ENGINEERING STRUCTURES AND TECHNOLOGIES
ISSN 2029-882X/eISSN 2029-8838
2014 6(1): 18–24
doi:10.3846/2029882X.2014.957901

EUROCODE STABILITY REQUIREMENTS IN OPTIMAL SHAKEDOWN TRUSS DESIGN

Gediminas BLAŽEVIČIUS, Juozas ATKOČIŪNAS

Vilnius Gediminas Technical University, Saulėtekio al. 11, LT-10223 Vilnius, Lithuania

Received 03 December 2013; accepted 01 June 2014

Abstract. The paper focuses on the optimization of a perfectly elastic-plastic truss under repeated variable load. The improved mathematical model of truss volume minimization problem with strength, stiffness and stability constraints is presented. The assumptions of the calculation methods of the truss-like structures and the shakedown theory are applied. The evaluation of the stability of elements under compression is based on EC3 requirements and related to plastic deformations in the shakedown process to correct the interpretation of the stability constraints in mathematical programming problems. In the optimization problem, truss displacements are evaluated according to different reliability levels of the ultimate and serviceability limit states of EC. The proposed methodology is illustrated with a numerical example. The results are valid for the assumption of small displacements.

Keywords: optimal shakedown design, elastic-plastic truss, standards, mathematical programming.

Introduction

To design more economical structures subjected to variable as well as repeated loading, the shakedown theory may be applied (Staat, Heitzer 2002; Weichert, Ponter 2009; Atkočiūnas 2011). This theory allows for the employment of the plastic properties of materials (particularly steel) for reducing the design structure's volume (mass). Though the process of shakedown is explored notionally in depth (Dang Van *et al.* 2002), it is still in the focus of researchers' and designers' interests (Tin-Loi 2000; Vu *et al.* 2007; Giambanco *et al.* 2012; Spiliopoulos, Panagiotou 2012). Practical structural design is always associated with national and international standards (Atkočiūnas, Venskus 2011). The Eurocode requirements (EN 1993-1-1 2005) allow for designing the structures with plastic deformations, though the optimization in the shakedown state has not been standardized. Therefore, in order to create a practically applicable mathematical model for the problem of truss volume minimization with strength, stiffness and stability constraints, it is necessary to correctly define the physical process of the shakedown (Cheng *et al.* 2012; Simon, Weichert 2012) and to assure that the structure should satisfy the requirements of the standards. The main problems associated with this task are considered in the current paper. First, the problem is associated with the application of the stress-strain dependence of the perfectly elastic-plastic truss to the bars under compression, which may potentially loose stability. The stability of bars is widely explored by many authors (Kaliszky, Lógó 2002; Ziemian 2010) and strictly regulated by the design standards. However, some problems of plastic state interpretation occur, when the algorithm of the stability check is implemented in the mathematical programming problem. It should be noted that the influence of bars under compression on the development of plastic deformations of a truss in the shakedown process cannot be interpreted in the same way as the influence of those under tension (by a formal explanation of a

Corresponding author:
G. Blaževičius E-mail: *gediminas.blazevicius@vgtu.lt*

yield condition satisfied as a strict equality). Second, the considered problem is associated with the displacements' constraints in the optimization problem of a perfectly elastic-plastic truss. Two different reliability levels for verification of the ultimate and serviceability limit states are used in the Eurocode and, in practical design, these limit states are usually evaluated separately. However, in searching for the optimal project of the structure, it is necessary to take into account both limit state requirements in solving one problem, i.e. to combine two different reliability levels in the same mathematical model. Therefore, a method of binary displacement calculation is proposed in this paper. The improved mathematical model of truss optimization, with the included strength, stiffness and stability constraints, is created. The new mathematical programming problem is non-convex due to the combinatorial complementary slackness conditions. The results of the numerical example of cantilever truss optimization are valid, when small displacement is assumed. This paper is based on the presentation given in an international conference (Atkočiūnas, Blaževičius 2012).

1. Mathematical model of optimal truss design

The numerical methods of structural mechanics are based on a discrete structural model. For this model, both general and particular mathematical models of problem solution (in our case, truss model) are developed. Dual relations between static (equilibrium) and kinematic (geometric) equations are taken into account, when choosing static and kinematic variables, which characterize the stress-strain state of the structure. The uniaxial stress state of a truss is expressed by the internal (axial) force vector $\boldsymbol{N} = [N_1 N_2 \ldots N_s]^T$, where s is the number of finite elements ($k = 1, 2, \ldots, s, k \in K$), constituting the discrete model. The variable repeated forces $\boldsymbol{F}(t)$, acting upon the elastic-plastic structure, are characterized by time-independent upper and lower bounds $\boldsymbol{F}_{\text{sup}}$, $\boldsymbol{F}_{\text{inf}}$. A detailed analysis of a loading history is omitted, when the loading is described by all possible combinations $\boldsymbol{F}_j$, $\boldsymbol{F}_{\text{inf}} \leq \boldsymbol{F}_j \leq \boldsymbol{F}_{\text{sup}}$, $j = 1, 2, \ldots, p, j \in J, p = 2^m$, where m is the number of the acting forces). These combinations can describe vertices of any loading locus or positions of loading when moving load is under consideration. The forces $\boldsymbol{N}_{ej}$ and the displacements $\boldsymbol{u}_{ej}$ of the elastic structure are determined, using the influence matrices of forces and displacements $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$: $\boldsymbol{N}_{ej} = \boldsymbol{\alpha F}_j$, $\boldsymbol{u}_{ej} = \boldsymbol{\beta F}_j$, $j \in J$. The limit force $N_{0,k}$ ($k \in K$) is assumed to be constant over the whole finite element k. Then, piecewise linearized yield conditions are $\boldsymbol{\Phi}(\boldsymbol{N}_r + \boldsymbol{N}_{ej} + \boldsymbol{N}_c) \leq \boldsymbol{N}_0$, $j \in J$, while $\boldsymbol{N}_r$ denotes the unknown statically admissible residual forces. The forces $\boldsymbol{N}_c$ are resulting from constant (invariable, permanent) loading: $\boldsymbol{N}_c = \boldsymbol{\alpha F}_c$. The optimization problem of the structure is stated as follows: *for the given load variation bounds $\boldsymbol{F}_{\text{sup}}$, $\boldsymbol{F}_{\text{inf}}$, the vector of the limit forces $\boldsymbol{N}_0$, satisfying the optimality criterion* $\min F(\boldsymbol{N}_0)$ *and the constraints of strength, stiffness and stability, should be found*:

find

$$\min F(\boldsymbol{N}_0), \tag{1}$$

subject to

$$\boldsymbol{\varphi}_{\max} = \boldsymbol{N}_0 - \boldsymbol{G\lambda} - \boldsymbol{N}_{e,\max} - \boldsymbol{N}_c \geq \boldsymbol{0}; \tag{2}$$

$$\boldsymbol{\varphi}_{\min} = \boldsymbol{N}_{0,cr} + \boldsymbol{G\lambda} + \boldsymbol{N}_{e,\min} - \boldsymbol{N}_c \geq \boldsymbol{0}; \tag{3}$$

$$\boldsymbol{\lambda}^T_{\max}\, \boldsymbol{\varphi}_{\max} = 0,\ \boldsymbol{\lambda}^T_{cr}\, \boldsymbol{\varphi}_{\min} = 0,\ \boldsymbol{\lambda} = [\boldsymbol{\lambda}_{\max}, \boldsymbol{\lambda}_{cr}] \geq \boldsymbol{0}; \tag{4}$$

$$\boldsymbol{N}_0 \geq \boldsymbol{N}_{0,\min}; \tag{5}$$

$$\boldsymbol{u}_{\text{inf}} \leq (\boldsymbol{H\lambda} + \boldsymbol{u}_{ej} + \boldsymbol{u}_c) \leq \boldsymbol{u}_{\text{sup}}; \tag{6}$$

$$j = 1, 2, \ldots, p, j \in J.$$

The objective function can implicitly express the minimum cost or the volume of the structure: $V = \boldsymbol{L}^T\boldsymbol{A}$, where $\boldsymbol{L}$ is the vector of the element length and $\boldsymbol{A}$ is the vector of the element's cross-sectional areas. The yield conditions (2)–(3) are written by implementing the vectors of the maximal and minimal values of the elastic axial forces $\boldsymbol{N}_{e,\max}$, $\boldsymbol{N}_{e,\min}$, such that $\boldsymbol{N}_{e,\min} \leq \boldsymbol{N}_{ej} = \boldsymbol{\alpha F}_j \leq \boldsymbol{N}_{e,\max}$, $j = 1, 2, \ldots, p, j \in J$. Then $\boldsymbol{\varphi}_{\max}$ and $\boldsymbol{\varphi}_{\min}$ are the vectors of the yield condition values of the elements under tension and compression, respectively. Yield conditions determine the vector of the statically admissible residual forces $\boldsymbol{N}_r = \boldsymbol{G\lambda}$, ensuring the shakedown of the elastic-plastic system under the given variable repeated load ($\boldsymbol{G}$ is the influence matrix of residual forces). The conditions (2)–(3), supplemented with the complementary slackness conditions of mathematical programming (4) ensure that the principle of minimum deformation energy of the unloaded system will be satisfied. Then, the components of the vector $\boldsymbol{\lambda}$ obtain the physical meaning of plastic multipliers (Zouain *et al.* 2002; Atkočiūnas, Venskus 2011). The displacements in the stiffness conditions (6) are as follows: the residual $\boldsymbol{u}_r = \boldsymbol{H\lambda}$, the elastic $\boldsymbol{u}_{ej}$ and $\boldsymbol{u}_c$, resulting from the invariable loading ($\boldsymbol{F}_c$). The limits of the displacements of the structure $\boldsymbol{u}_{\text{inf}}$ and $\boldsymbol{u}_{\text{sup}}$ are determined according to the Eurocode requirements. The limit axial force of the k-th element under tension is calculated basically as the product of the cross-sectional area and yield stress: $N_{0,k} = A_k \cdot f_y$, whereas the limit axial force of the element under tension must be reduced because of a possible loss of stability. The Eurocode methodology of reducing the limit axial force of an element under compression will be considered in this paper. It states that for the k-th discrete element $N_{0,cr,k} = \chi_k \cdot N_{0,k}$, while the reduction coefficient χ is the function of the element's geometrical and physical characteristics. The vector of the limit forces $\boldsymbol{N}_0$ and the vector of plastic multipliers $\boldsymbol{\lambda}$ are the unknowns in the problem (1)–(6). This optimization problem is non-convex due to the combinatorial complementary slackness conditions. Taking into account that the problem conditions depend on the unknowns, the solution algorithm is iterative. Similar shakedown design method, with the stability evaluation and the assumption that $\boldsymbol{N}_c = 0$ and $\boldsymbol{u}_c = 0$, was used in the previous publication (Merkevičiūtė, Atkočiūnas 2006).

1.1. Plastic deformations under stability conditions

When mathematical programming is used for optimal shakedown truss design, the complementary slackness conditions of mathematical programming (4) are written down alongside strength conditions. The multipliers $\boldsymbol{\lambda} = [\boldsymbol{\lambda}_{\max}, \boldsymbol{\lambda}_{cr}]$ obtain the physical meaning of plastic multipliers for the elements under tension and compression, respectively. In designing the elastic-plastic bar structures, the stress-strain state is usually simplified, using the so-called Prandtl diagram. It is further used to explain the emergence of plastic deformations in the shakedown process (Fig. 1). When a positive side of the graph (positive stress f and strain ε) referring to the elements under tension is considered, it is evident that plastic deformations occur only when the elastic state (the section 0-A) is over, when the stress reaches the yield stress value, i.e. $f = f_y$ (the section A-B). A more complicated case is found, when the elements under compression are examined. In the simplest case, when the element's buckling is not considered, a negative side of the graph is symmetric to the positive one, i.e. the element is deformed according to the curve 0-D-E. The same case is found, when the critical stress reaches the yield stress $f_{cr} = f_y$. According to the Eurocode, such case refers to the elements with very small non-dimensional slenderness: $\bar{\lambda} \leq 0.2$ (it should not be confused with plastic multipliers λ).

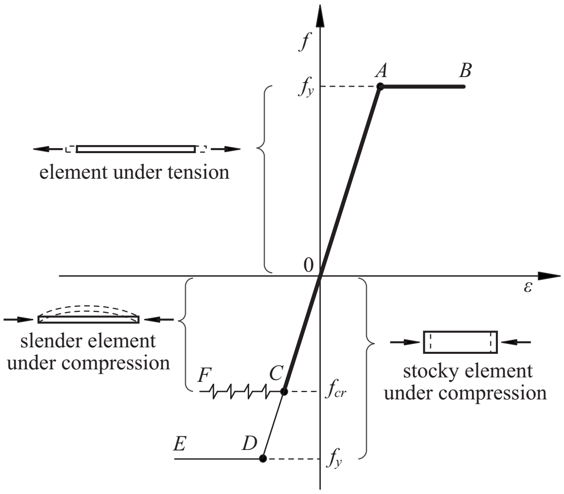

Fig. 1. Stress-strain graph of perfectly elastic-plastic material

When stability verification is implemented in the mathematical programming problem, it is found that, in the general case, the deformations emerge according to the curve 0-C-F. However, contrary to the case of tension, the plastic deformations of the elements under compression (when the limit state is reached, i.e. after the loss of stability) are not defined in the EC and cannot be evaluated. Therefore, a true deformation curve of the element under compression is only elastic – 0-C, if $C \neq D$, or elastic-plastic – 0-D-E, when $D = C$ and $E = F$. Thus, the solution algorithm of the mathematical programming problem comes into conflict with the Eurocode requirements. Therefore, the above-mentioned complementary slackness conditions (4) are inadequate for ensuring the shakedown of a truss. This inaccuracy is eliminated by introducing a new condition in the mathematical model, which ensures that plastic multipliers (i.e. plastic deformations) can emerge only due to the limit stress of the elements under tension or in very stocky elements (small non-dimensional slenderness) under compression:

$$\lambda_{cr,k} (N_{0,k} - N_{0,cr,k}) = 0, k = 1, 2, ..., s, k \in K. \quad (7)$$

This condition ensures that slender elements under compression (when $N_{0,cr,k} \leq N_{0,k}$, $\chi \leq 1$) cannon cause the occurrence of nonzero plastic multipliers. The correct determination of the plastic multipliers $\boldsymbol{\lambda}$ is an essential task because they are used in the same problem for calculating the residual forces and displacements.

1.2. Displacement constraints according to the Eurocode

In the Eurocode standards, all design calculations are divided into two groups and are aimed at verifying the ultimate and serviceability limit states. Two different reliability levels are used for these limit states. In using the partial factor method, these levels are achieved by applying the respective representative values of the action. When the strength (2) and stability (3) conditions of the mathematical model are there for the ultimate limit state verification, the serviceability limit state for the structure must be secured as well. A structure can be reliable only if none of the limit states is exceeded. Therefore, stiffness conditions (6) (displacement constraints of the truss nodes) must be introduced into the model. Regarding Eurocode they can be specified as follows:

$$\boldsymbol{u}_{\inf} \leq \left(\boldsymbol{u}_r + \overline{\boldsymbol{u}}_{ej} + \overline{\boldsymbol{u}}_c\right) \leq \boldsymbol{u}_{\sup}, j = 1, 2, ..., p, j \in J, \quad (8)$$

where $\boldsymbol{u}_{\sup}$ and $\boldsymbol{u}_{\inf}$ are the known vectors of the upper and lower admissible bounds of displacement variation. The displacement of a perfectly elastic-plastic truss consists of two components: the residual $\boldsymbol{u}_r = \boldsymbol{H\lambda}$ and pseudo-elastic $\overline{\boldsymbol{u}}_{ej} + \overline{\boldsymbol{u}}_c$. The residual component is obtained from the shakedown process (by using the plastic multipliers $\boldsymbol{\lambda}$ and the influence matrix $\boldsymbol{H}$); therefore, it is determined by the ultimate state with a high reliability level. The vector $\boldsymbol{u}_r$ is calculated in the optimization process of a structure subjected to variable repeated loading of design values. Therefore, for all possible combinations of loading j, $\boldsymbol{F}_{jd} = \gamma_E \cdot \boldsymbol{F}_{jk}$ (where index d means the *design value*, k is the *characteristic value* and γ_E is a partial factor for the action). The pseudo-elastic component is calculated, using the Hooke's law, and determined by the serviceability limit state with a lower reliability level. It is calculated, using all possible combinations of the characteristic loading values:

$$\overline{\boldsymbol{u}}_{ej} + \overline{\boldsymbol{u}}_c = \boldsymbol{\beta F}_{jk} + \boldsymbol{\beta F}_{ck}, \; j = 1, 2, ..., p, j \in J. \quad (9)$$

This approach, based on the dual reliability level, allows for designing a more economical structure, compared to the earlier presented models because lower reliability is used for the displacement constraints than the strength and stability conditions.

1.3. The improved model of truss volume optimization

The improved mathematical model of the problem of volume minimization of a perfectly elastic-plastic truss with new displacement constraints and plastic deformation conditions can be expressed as follows:

find

$$\min \boldsymbol{L}^T \boldsymbol{A}, \quad (10)$$

subject to

$$\boldsymbol{\varphi}_{\max} = \boldsymbol{N}_0 - \boldsymbol{G\lambda} - \boldsymbol{N}_{e,\max} \geq \boldsymbol{0}; \quad (11)$$

$$\boldsymbol{\varphi}_{\min} = \boldsymbol{N}_{0,cr} + \boldsymbol{G\lambda} + \boldsymbol{N}_{e,\min} \geq \boldsymbol{0}; \quad (12)$$

$$\boldsymbol{\lambda}^T_{\max} \boldsymbol{\varphi}_{\max} = 0, \boldsymbol{\lambda}^T_{cr} \boldsymbol{\varphi}_{\min} = 0, \boldsymbol{\lambda} = [\boldsymbol{\lambda}_{\max}, \boldsymbol{\lambda}_{cr}] \geq \boldsymbol{0}; \quad (13)$$

$$\lambda_{cr,k} (N_{0,k} - N_{0,cr,k}) = 0, k = 1, 2, ..., s, k \in K; \quad (14)$$

$$\boldsymbol{A} \geq \boldsymbol{A}_{\min}; \quad (15)$$

$$\boldsymbol{u}_{\inf} \leq \left(\boldsymbol{u}_r + \overline{\boldsymbol{u}}_{ej} + \overline{\boldsymbol{u}}_c\right) \leq \boldsymbol{u}_{\sup}, \; j = 1, ..., p, \; j \in J. \quad (16)$$

The model consist of the yield conditions (11)–(12), the complementary slackness conditions of mathematical programming (13), the complementary conditions for plastic multipliers (14), the construction regulation constraints (15) and the displacement constraints (16). $\boldsymbol{A}_{\min}$ is the vector of the minimum values of cross-sectional areas of all elements (usually, it is determined by joint construction or other design requirements). The unknowns of the problem (10)–(16) are the vector of the element's cross-sectional areas $\boldsymbol{A}$ and the vector of plastic multipliers $\boldsymbol{\lambda}$. The influential matrices of the elastic forces, elastic displacements, residual forces and residual displacements $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{G}$, $\boldsymbol{H}$, used for calculations, are dependent on the variable cross-sectional areas $\boldsymbol{A}$. Therefore, the solution algorithm is performed in an iterative manner (Fig. 2).

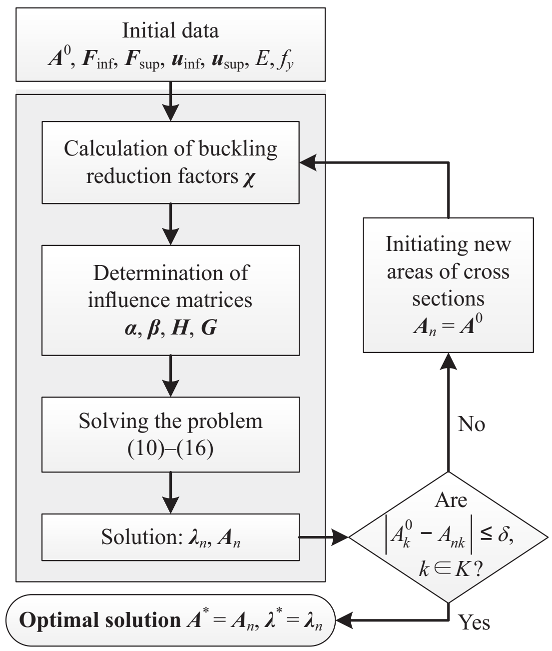

Fig. 2. Flowchart of the proposed solution algorithm

This model suits for either – discrete or continuous – optimization. The discrete optimization is more practical, but, the continuous optimization can also be used. For example, using section properties, obtained from the continuous optimization, it is possible to choose nearest fitting discrete cross-section from assortment.

2. Numerical example

The minimum volume problem of the cantilever truss with parabolic compression chord, shown in Figure 3, is considered. The truss is subjected to the action of permanent (constant) $\boldsymbol{F}_c$ and moving (variable) $\boldsymbol{F}_j$ loads (characteristic values in kN are given in the Fig. 3). Following partial factors are used for the design values of the loads: $\lambda_{Gj,\text{sup}} = 1.35$ for the moving load and $\lambda_{Q,1} = 1.3$ for the permanent load. The main task is to solve the problem (10)–(16), i.e. to determine the cross-sectional areas $\boldsymbol{A}$ of the elements and the volume V of the whole structure. The bars of the truss are grouped into four groups: compression (bottom) chord A_1, tension (top) chord A_2, vertical web A_3 and inclined web A_4. The prescribed minimum values of the cross-sectional areas are equal to $A_{1,\min} = A_{2,\min} = A_{3,\min} = A_{4,\min} = 0.006$ m^2. The sections of the bars are square hollow (SHS) with selected heights of $b_1 = 16$ cm, $b_2 = 14$ cm, $b_3 = b_4 = 10$ cm. These values can be changed according to the optimization results. The elasticity modulus of the material is $E = 210$ GPa, while the yield stress $f_y = 235$ MPa.

The presented method of binary displacement calculation is used in both cases. The following total displacement constraint is imposed: $u_v \leq 35$ mm. In order to evaluate the changes made in the new mathematical model, two different cases of truss optimization are considered:

Case C1 – truss volume optimization using the classical model (1)–(6);

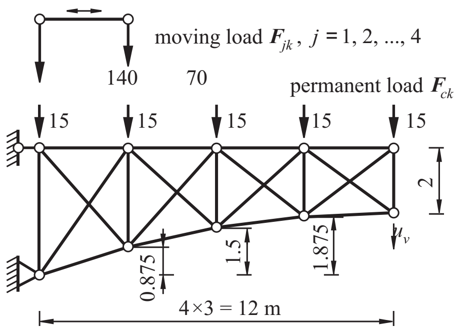

Fig. 3. Geometry and loading of the truss

Case C2 – truss volume optimization using the improved model with new complementary conditions for plastic multipliers (10)–(16).

The results of the numerical optimization calculations are shown in the Table 1. Truss plastic deformations in the particular optimization cases are illustrated in Fig. 4. Optimal truss volume values were found: $V = 0.15916$ m^3 in the case C1, and $V = 0.17269$ m^3 in the case C2.

Table 1. Optimization results

Element		Bottom chord	Top chord	Vertical web	Inclined web
C1	A, m^2	39.068	29.494	20.975	14.556
	t, mm	6.43	5.55	5.64	3.83
C2	A, m^2	37.730	30.572	18.264	19.982
	t, mm	6.20	5.76	4.87	5.36

The complementary conditions for plastic multipliers (14) in the case C2 did not allow to emerge plastic deformations in the elements under compression as it was in the case C1. Thus different stress state and different distribution of residual forces and plastic deformations are found. Bigger value of the truss volume is found in the case C2. The distribution of the cross-sectional areas also differs. The optimal cross-sectional areas of the bottom chord and vertical web elements are found to be smaller, meanwhile cross-sectional areas of the top chord and inclined web elements – bigger in the case C2.

Constraint of the node displacement is satisfied as strict equality in both cases. However, due to reduced possibility of plastic deformations emergence, heavier but in also more buckling resistant structure was designed using the new optimization model (10)–(16).

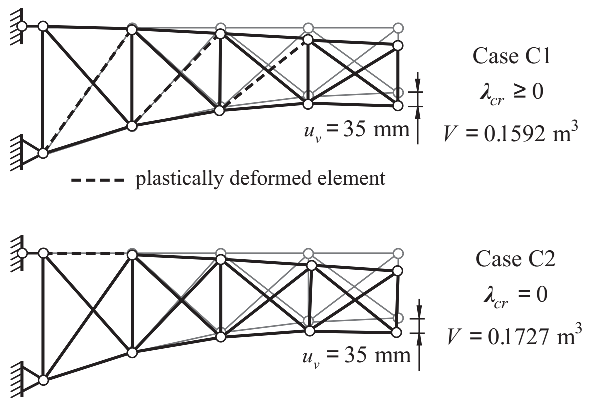

Fig. 4. Truss deformations and the optimal volumes

Conclusions

Practical implementation of optimal shakedown design should not be based only on theoretical improvements, but should take into account the existing design standards. When the stability requirements are implemented in a mathematical programming problem, some difficulties in evaluating plastic multipliers arise. The complementary slackness conditions are not adequate to ensure the shakedown state. The improved mathematical model of truss volume minimization problem with strength, stiffness and stability constraints is presented. Different reliability levels are used for the verification of the ultimate and serviceability limit states of trusses in the suggested mathematical model. Thus, the shakedown theory acquires the potentiality to be used in actual standardized truss design.

References

Atkočiūnas, J.; Blaževičius, G. 2012. Eurocode stability requirements in optimal shakedown truss design, in *Topping BHV (Ed). Proceedings of the Eleventh International Conference on Computational Structures Technology*, Paper 268, 1–11. Stirlingshire UK: Civil-Comp Press. http://dx.doi.org/10.4203/ccp.99.268

Atkočiūnas, J. 2011. *Optimal shakedown design of elastic-plastic structures*. Vilnius: Technika. 300 p. http://dx.doi.org/10.3846/1240-S

Atkočiūnas, J.; Venskus, A. 2011. Optimal shakedown design of frames under stability conditions according to standards, *Computers & Structures* 89(3–4): 435–443. http://dx.doi.org/10.1016/j.compstruc.2010.11.014

Cheng, L.; Jia, Y.; Oueslati, A.; De Saxcé, G.; Kondo, D. 2012. Plastic limit state of the hollow sphere model with non-associated Drucker–Prager material under isotropic loading, *Computational Materials Science* 62: 210–215. http://dx.doi.org/10.1016/j.commatsci.2012.05.048

Dang Van, K., *et al.* 2002. *Inelastic behaviour of structures under variable repeated loads*. Wien: Springer. 393 p.

Giambanco, F.; Benfratello, S.; Palizzolo, L.; Tabbuso, P. 2012. Structural design of frames able to prevent element buckling, in *Topping BHV (Ed.). Proceedings of the Eleventh International Conference on Computational Structures Technology*, Paper 18, 1–16. Stirlingshire UK: Civil-Comp Press. http://dx.doi.org/10.4203/ccp.99.18

Kaliszky, S.; Lógó, J. 2002. Plastic behaviour and stability constraints in the shakedown analysis and optimal design of trusses, *Structural and Multidisciplinary Optimization* 24(2): 118–124. http://dx.doi.org/10.1007/s00158-002-0222-2

Merkevičiūtė, D.; Atkočiūnas, J. 2006. Optimal shakedown design of metal structures under stiffness and stability constraints, *Journal of Constructional Steel Research* 62(12): 1270–1275. http://dx.doi.org/10.1016/j.jcsr.2006.04.020

Simon, J. W.; Weichert, D. 2012. Shakedown analysis of engineering structures with limited kinematical hardening, *International Journal of Solids and Structures* 49(15–16): 2177–2186. http://dx.doi.org/10.1016/j.ijsolstr.2012.04.039

Spiliopoulos, K.V.; Panagiotou, K. D. 2012. A direct method to predict cyclic steady states of elastoplastic structures, *Computer Methods in Applied Mechanics and Engineering* 223–224: 186–198. http://dx.doi.org/10.1016/j.cma.2012.03.004

Staat, M.; Heitzer, M. (Eds.). 2002. *Numerical methods for limit and shakedown analysis – deterministic and probabilistic problems*. John von Neumann Institute for Computing.

EN 1993-1-1: 2005. Design of steel structures Part 1-1: General rules and rules for buildings. Brussels, 2005.

Tin-Loi, F. 2000. Optimum shakedown design under residual displacement constraints, *Structural and Multidisciplinary Optimization* 19(2): 130–139. http://dx.doi.org/10.1007/s001580050093

Vu, D. K.; Staat, M.; Tran, I. T. 2007. Analysis of pressure equipment by application of the primal-dual theory of shakedown, *Communications in Numerical Methods in Engineering* 23(3): 213–225. http://dx.doi.org/10.1002/cnm.891

Weichert, D.; Ponter, A. (Eds.). 2009. *Limit states of materials and structures*. Dordrecht: Springer Netherlands. http://dx.doi.org/10.1007/978-1-4020-9634-1

Ziemian, R. D. (Ed.) 2010. *Guide to stability design criteria for metal structures*. 6th ed. Wiley. 1024 p. http://dx.doi.org/10.1002/9780470549087

Zouain, N.; Borges, L.; Luís Silveira, J. 2002. An algorithm for shakedown analysis with nonlinear yield functions, *Computer Methods in Applied Mechanics and Engineering* 191(23–24): 2463–2481. http://dx.doi.org/10.1016/S0045-7825(01)00374-7

OPTIMALIOS PRISITAIKANČIOS SANTVAROS PROJEKTAVIMAS TAIKANT EUROKODO REIKALAVIMUS

G. Blaževičius, J. Atkočiūnas

Santrauka. Straipsnyje nagrinėjamas idealiai tamprios ir plastinės santvaros, veikiamos kintamosios kartotinės apkrovos, optimizavimas. Taikomos santvarinių konstrukcijų skaičiavimo techninės ir prisitaikymo teorijos prielaidos. Sudarytas pagerintas santvaros tūrio minimizavimo uždavinio matematinis modelis su stiprumo, standumo ir stabilumo apribojimais. Konstrukcijos gniuždomųjų elementų galimas stabilumo praradimas tikrinamas pagal Europos projektavimo normų (EN) reikalavimus, siejamus su prisitaikymo proceso plastinėmis deformacijomis. Straipsnyje pateikiamos naujos sąlygos, papildančios optimizavimo uždavinį ir patikslinančios stabilumo apribojimų interpretaciją prisitaikymo procese. Skirtingai nei tempiamųjų, gniuždomųjų strypų plastinės deformacijos (viršijus saugos ribinį būvį, t. y. išklupus) neapibrėžtos EN ir nėra vertinamos. Tad klasikinės matematinio programavimo griežtumo sąlygos yra nepakankamos idealiai tamprios plastinės santvaros prisitaikymui užtikrinti. Šis netikslumas pašalinamas pritaikius naują sąlygą, užtikrinančią, kad nenuliniai plastiniai daugikliai gali atsirasti tik dėl tempiamųjų strypų arba gniuždomųjų labai tvirtų (mažo sąlyginio liaunio) strypų takumo įtempių. Santvaros įlinkiai tūrio minimizavimo uždaviniuose ribojami atsižvelgiant į EN saugos ir tinkamumo ribinių būvių patikimumo lygmenis. Straipsnyje pristatoma metodika, kurioje liekamoji poslinkio dalis gaunama iš prisitaikymo proceso, taigi yra nulemta saugos ribinio būvio ir sąlygiškai aukštesnio patikimumo. Tariamai tamprioji poslinkio dalis skaičiuojama pagal Huko dėsnį ir yra formuojama tinkamumo ribinio būvio. Toks dviejų patikimumo lygių taikymas pagrįstas EN reikalavimais ir leidžia projektuoti ekonomiškesnę konstrukciją, palyginti su ankstesnių tyrėjų pasiūlytais modeliais. Metodika iliustruojama skaitiniu pavyzdžiu, taikant mažų poslinkių prielaidą.

Reikšminiai žodžiai: optimalus projektavimas, prisitaikymas, santvara, standartai, matematinis programavimas.

Gediminas BLAŽEVIČIUS. PhD student at the Department of Structural Mechanics in Vilnius Gediminas Technical University. Research interests: structural optimization, optimal shakedown design of structures.

Juozas ATKOČIŪNAS. Prof. Dr Habil, Head of the Department of Structural Mechanics in Vilnius Gediminas Technical University. Research interests: computational mechanics, shakedown theory of elastic-plastic structures, structural optimization and methods of mathematical programming in structural mechanics.