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DISPLACEMENT ANALYSIS OF ASYMMETRICALLY LOADED CABLE

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Abstract. A deformed state of parabolic shape suspension cable subjected to asymmetric loading is under consideration. The cable kinematic nature of vertical and horizontal displacements, resulted from complementary asymmetrically distributed load are investigated, the expressions for their maximum values are derived. The deformability of suspension cable versus its curvature increment is investigated. The possibilities to stabilise the primary parabolic shape of suspension cable subjected to an asymmetric load are examined. Numerical simulation results are presented.

Keywords: cable structure, asymmetric loading, non-linear analysis, kinematic displacements.

1. Introduction

The suspension flexible cable as the main carrying member of complex structure is successfully applied in design of large span bridges, roofs of various buildings [1–12]. A long-term service experience has shown that idiosyncratic and unwished feature of the loaded suspension cable is a large deformability [4, 6, 8, 9, 12, 13]. The deformability is conditioned by the appearance of the elastic and non-straining (kinematic) displacements. The elastic displacements are caused by large tensile inner forces, resulted from maximal symmetric loads and those of inelastic (kinematic) kind – by primary parabolic shape cable changes, resulted from asymmetric or local loads [4–9,13–18]. In actual design of structures the deformability limit state constraints become dominating:

$$\omega_{max} \le \omega_{lim}, \tag{1}$$

$$\omega''_{max} \le \omega''_{lim}, \tag{2}$$

where ω_{max}, ω''_{max} are the the maximal displacement and its second derivative; ω_{lim}, ω''_{lim} are the maximal admitted displacement and its admitted second derivative (curvature change).

The accuracy, when evaluating ω_{max}, ω''_{max} in design process, directly influences the cable structures, supports both coverings maintenance reliability and their technical-economical efficiency. One can list many investigations devoted to suspension cable deformable behaviour analysis and its general (elastic and kinematic) displacements evaluations [4, 8, 13, 15–23]. One must note that the simplified engineering methods are most often employed to evaluate the general (total) vertical displacements of suspension cable [4, 7, 13–15, 17, 18]. The latter methods are based on superposition principle, when splitting the actual loads to the symmetric and asymmetric ones. The superposition principle employed for suspension cable, responding to loading non-linearly, results in certain mistakes when evaluating its strain state. The investigation [17] is referred to the estimation of errors, that appear when calculating the general displacements of suspension cable subjected to an asymmetric load. The equivalent symmetric load concept is proposed to reduce the error in the investigation [4, 19]. One must note that the list of investigations devoted to kinematic displacements analysis of asymmetrically loaded suspension cable is small enough. The more exact kinematic displacements evaluation methods should give closer to the actual behaviour results as well as to help clarify the application bounds of simplified engineering methods.

The present investigation is devoted to the development of suspension parabolic shape cable kinematic vertical and horizontal displacement calculation methods. The expressions for maximal displacement magnitudes are derived. The possibilities to stabilise the primary parabolic shape of suspension cable subjected by asymmetric load are examined. The analysis of the above cable displacement evaluation errors, obtained by employing the widely applied engineering methods, is provided. A comparison of displacement numerical evaluation by engineering and that of proposed methods is presented.

2. Deformed state of asymmetrically loaded flexible suspension cable

Consider the deformed state of an inelastic flexible suspension cable subjected to the primary constantly distributed load g and the supplementary constantly distributed load p, applied to the left half span (Fig 1). The considerations are to be provided in respect of the element inelastic kinematic displacements, ie taking the elastic displacements to be negligibly small $(EA \to \infty)$

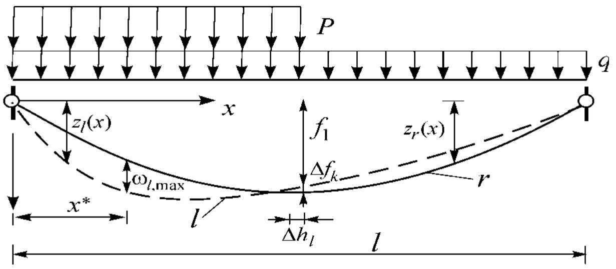

Fig 1. Cable vertical and horizontal displacements versus asymmetric load p

The primary geometrical shape of the structure, subjected to loading q corresponds to quadratic parabola, the primary sag being f_0 in the half span. The application of the supplementary asymmetric load p changes the equilibrium form.

The primary shape function of the suspension cable subjected by the load p corresponds to the quadratic parabola equation:

$$z(x) = \frac{M_0(x)}{H_0} = f_0\left[\left(\frac{4x}{l}\right) - \left(\frac{4x^2}{l^2}\right)\right], \tag{3}$$

where $M_0(x)$ – a bending moment resulting from the symmetric load q, H_0 – the thrusting (tensile) inner force of the cable.

Divide artificially the cable into the left part subjected to the complementary load p and the right part subjected only to the primary load q. Then the deformed cable axis function can be described by the equation [24]:

when $x \le l/2$,

$$z_l(x) = \frac{M_l(x)}{H_{l1}} = \frac{f_1}{\left(1+\frac{\gamma}{2}\right)} \cdot \left[\left(\frac{4x}{l} - \frac{4x^2}{l^2}\right) + \gamma\left(\frac{3x}{l} - \frac{4x^2}{l^2}\right)\right]; \tag{4}$$

when $l/2 \le x \le l$,

$$z_r(x) = \frac{M_r(x)}{H_{r1}} = f_1\left[\left(\frac{4x}{l} - \frac{4x^2}{l^2}\right) + \gamma\left(\frac{x}{l} - 1\right)\right] \Big/ \left(1+\frac{\gamma}{2}\right), \tag{5}$$

where $M_{l1}(x)$, H_{l1} are the bending moments and the tensile force of the left loaded by $(q+p)$ cable part, respectively; $M_{r1}(x)$, H_{r1} are the bending moments and the thrusting force of the cable right part, respectively; $f_1 = f_0 + \Delta f_k$ is the cable deflection in the middle span ($x = l/2$) after application of the supplementary load; Δf_k is the inelastic (kinematic) cable middle span deflection (Fig 1); $\gamma = p/q$ is the ratio of asymmetric and symmetric loads intensities.

An analysis of the equations (4) and (5) shows the axial curvature function of the cable left part to be the "sum" of two parabola functions, the cable left part curvature function corresponding to the "sum" of parabola and line functions. The maximal deflection location is identified applying the deflection function $z_l'(x) = 0$ derivative:

$$x^* = \frac{l}{4} \cdot \frac{(2+3\gamma/2)}{(1+\gamma)}. \tag{6}$$

Analysing the expression (6) one can find the maximal deflection location to be dependent on the loads ratio γ. Varying the γ magnitude from 1 to 10, the maximal deflection location varies in inside of bounds $x^* = 0{,}437l - 0{,}386l$. The result proves the maximal deflection to be insignificantly deviated from the middle span.

3. Cable left part vertical kinematic displacements

In order to satisfy the design requirements (1), (2) when designing the cable structures, one must identify the maximal general (total) as well as its very significant component – kinematic vertical displacements. The latter for cable left part can be determined by the following expression:

$$\omega_l(x) = z_{l1}(x) - z_{l0}(x), \tag{7}$$

where $z_{l0}(x)$ – cable primary shape function; $z_{l1}(x)$ – cable left part shape function after application of load p.

Applying (7) and combining (3) and (4), one obtains:

$$\omega_l(x) = \frac{f_1}{(1+\gamma/2)}\left[\left(\frac{4x}{l} - \frac{4x^2}{l^2}\right) + \gamma\left(\frac{3x}{l} - \frac{4x^2}{l^2}\right)\right] - f_0\left(\frac{4x}{l} - \frac{4x^2}{l^2}\right) \tag{8}$$

It is obvious that for the middle span ($x \le l/2$)

$$\omega_l(x = l/2) = f_0 - f_1 = \Delta f_k . \tag{9}$$

An analysis of (8) and (9) expressions concludes that the kinematic displacements $w_l(x)$ can be identified only when f_1 or Δf_k are already known. To identify the latter values, apply the deformed suspension cable length expression:

$$S_1 = S_l + S_r \cong l + 0{,}5\int_0^{l/z}[z_l'(x)]^2dx + 0{,}5\int_{l/2}^{l}[z_r'(x)]^2dx. \quad (10)$$

Having solved the equation (10) and combining (3) and (4), one obtains:

$$S_1 = l + \frac{8}{3}\frac{f_1^2}{l}\psi, \quad (11)$$

where

$$\psi = \frac{1+\gamma+\gamma^2/4}{1+\gamma+5\gamma^2/16}. \quad (12)$$

Write the inelastic cable $(EA \to \infty)$ length expressions $S_1 = S_0$ for its states before and after loading by p. The latter expressions allow to identify the cable middle span deflection:

$$f_1 = f_0\sqrt{\psi}. \quad (13)$$

From (12) and (13) relations analysis one can obviously find $\psi \le 1{,}0$, meaning that $f_1 < f_0$.

Employing the f_1 expression, one can find the cable kinematic (inelastic) middle span displacement:

$$\Delta f_k = f_0\left(\sqrt{\psi}-1\right). \quad (14)$$

From expression (14), taking into account (12), one can find the value Δf_k always to be of a negative magnitude. Thus, one can conclude that the suspension cable responses to asymmetric load by middle span displacement directed up, and this lifting displacement magnitude is in direct proportion to loads ratio γ magnitude. For example, when $\gamma = 1$, $\Delta f_k = -0{,}0136 f_0$, when $\gamma = 3$, $\Delta f_k = -0{,}0422 f_0$. The absolute Δf_k magnitude is in direct proportion to the cable sag f_0.

One must note that engineering cable analysis methods employing the superposition principle lead to the zero magnitude of kinematic displacement independently from γ magnitude [4, 7, 13, 15, 19].

Having identified the cable deflection f_1, one can determine its left part vertical displacements by:

$$\omega_l(x) = f_0\left[\left(\frac{4x}{l}-\frac{4x^2}{l^2}\right)\left(\frac{1}{\xi}-1\right)+\frac{\gamma}{\xi}\left(\frac{3x}{l}-\frac{4x^2}{l^2}\right)\right], \quad (15)$$

where

$$\xi = \sqrt{1+\gamma+5\gamma^2/16}. \quad (16)$$

As one can find from (15), the kinematic displacements are directly dependent on the cable sag. The increase of the loads ratio γ results in the increase of the $\omega_l(x)$ magnitudes. The obtained expression (16) is convenient for usage, as it does not include f_1. It is obvious that kinematic displacement in the middle span ($x \le l/2$) is $\omega_l(x) = \Delta f_k$.

In practical design one must identify maximal deflection and its location point. This point can be identified having equaled to zero the deflection function first derivative ($\omega_l'(x) = 0$). Then the distance from the left support to the maximal deflection point is:

$$x^{**} = \frac{l}{4}\frac{(2+3\gamma/2-2\xi)}{(1+\gamma-\xi)}. \quad (17)$$

The analysis expression (17) proves the maximal deflection of the cable left part to be outside of the fourth quarter $x^{**} = l/4$, in contradiction to [4–8, 13–19]. When increasing the loads ratio γ, the latter distance decreases. By applying the expression (17) it was proved that increasing the loads ratio γ from 1 to 10, the maximal deflection location point varies insignificantly, ie $x^{**} = (0{,}9568 - 0{,}889)l/4$. One must note that size of maximal displacements calculations via relations (15) and (17) are of a rather large size. Taking $x^{**} = l/4$ from (15), one obtains the approximate formula for loaded part displacement evaluation:

$$\omega_l(x) = \frac{3}{4}f_0\left[\frac{(1+2\gamma/3)}{\xi}-1\right]. \quad (18)$$

The above formula is rather compact and does not require complicated and large calculations. Analysis of the formula (18) proved that it produces insignificant errors when compared with the formulae (16) and (17), for instance: 0,14 %, when $\gamma = 1$; 1,56 % when $\gamma = 10$.

4. Cable right part vertical kinematic displacements

Kinematic displacements of the cable right part, being free of loading, were under investigation in [4–8, 13–19], stating them to be of absolute magnitude as these of the cable loaded part.

Let us identify the cable unloaded part vertical displacements employing the expression analogous to (7):

$$\omega_r(x) = z_{r1}(x) - z_{r0}(x), \quad (19)$$

where $z_{r0}(x)$ – the primary cable shape function; $z_{r1}(x)$ – the cable right part shape function after application of the asymmetric supplement load p.

The general solution of the expressions, (3) and (5), (19) results in displacements calculation formula:

$$\omega_r(x) = \frac{f_1}{(1+\gamma/2)}\left[\left(\frac{4x}{l}-\frac{4x^2}{l^2}\right)+\gamma\left(1-\frac{x}{l}\right)\right] - f_0\left(\frac{4x}{l}-\frac{4x^2}{l^2}\right) \quad (20)$$

Taking into account (13), one can derive from (20):

$$\omega_r(x) = f_0\left[\left(\frac{4x}{l}-\frac{4x^2}{l^2}\right)\left(\frac{1}{\xi}-1\right)+\frac{\gamma}{\xi}\left(1-\frac{x}{l}\right)\right]. \quad (21)$$

The formula (21) is analogous to the (15) expression. One can find from relation (21) that the vertical kinematic displacements of the unloaded right part are in direct proportion to the cable sag and the load ratio γ.

Having equalled the first derivative of the expression (21) to zero, one can identify the location point of the maximal displacement of the right part:

$$x^{**} = \frac{3l}{4}\frac{(\xi - \gamma/4 - 1)}{(\xi - 1)}. \tag{22}$$

Analysis of the expression (22) shows that varying γ from 1 to 10, the x^{**} variation bounds are close enough. As $\omega_r(x^{**})$ magnitudes inside of the above bounds differ insignificantly, we take them to be analogous to w of the left part, ie $\omega_{r,\max} = \omega_r(x^{**} = 3l/4)$. Then from equation (21) one can obtain the simplified formula to calculate the maximal displacements of the considered cable part:

$$\omega_l(x) = \frac{3}{4} f_0\left[\left(\frac{1}{\xi} - 1\right) + \frac{\gamma}{3\xi}\right]. \tag{23}$$

The formula (23) application error does not exceed 2 % considered γ variation range when compared with the formula (21).

Analysing the left part displacement $\omega_{l,\max}$ and that of the right part $\omega_{r,\max}$, one can find that the unloaded part displacements are larger in absolute magnitudes when compared with the ones of the cable loaded part. It is proved that when increasing the loads ratio γ – the difference between these values in absolute magnitudes increases as well. One can state that the unloaded cable part displacements are the governing ones, when designing the cable structures in respect of the stiffness requirements (limit state conditions). Besides, the negative cable displacements can be dangerous for floorings or partitions.

5. Curvature change of asymmetrically loaded cable

The main maintenance requirement of suspension cable, being the main carrying structural element, is the primary equilibrium form stability. It is the value being an inverse to deformability. Cable displacements stabilisation according to (1) is necessary for usual rigid structures too, but the limitation of the load increments due to (2) is applied for more "flexible" structures. The suspension cable reacts to the asymmetric load by its shape change, leading to significant curvature changes. The latter influence is directly connected with the continuity and tightness of floorings and partitions. Although structural design codes, valid in Lithuania [25], do not regulate admissible curvature increments, these magnitudes can be obtained when analysing the known construction of flooring and partitions by theoretical or experimental methods [26].

The curvature equation for cable part subjected by supplement asymmetric load can be obtained from the expression (4):

$$\omega''_l = \frac{8 f_0}{l^2}\left[1 - \frac{1+\gamma}{\xi}\right]. \tag{24}$$

Analysing the formula (24), one can find the curvature change to be dependent on the cable sag and span magnitude as well as on the asymmetric loads intensities ratio. A larger asymmetric load induces larger curvature increment in the load activity zone.

The right unloaded part cable curvature change is calculated by the formula:

$$\omega''_r = \frac{8 f_0}{l^2}\left[1 - \frac{1}{\xi}\right]. \tag{25}$$

An analysis of the unloaded cable part curvature changes shows that the relative increase of asymmetric load the cable results in the "straightening" of this cable part. Having compared the curvature changes of the loaded and unloaded cable parts, one can obviously find the curvatures to be larger in absolute magnitudes in the unloaded cable part, in contradiction as stated to be equal ones in the known investigations [4–8, 13–19].

6. Horizontal kinematic displacements

The induced by asymmetric load vertical kinematic displacements are accompanied by the horizontal ones. Consider separately the horizontal displacements of the left and right cable parts.

Identify the horizontal displacement of the left part at the middle span ($x = l/2$) point applying the following simplified expression:

$$\Delta h_l = (S_{l1} - S_{l0}), \tag{26}$$

where S_{l0}, S_{l1} – the cable left part lengths prior and after the deformation (the loading by load *p*), respectively.

Combining the (3), (4) and (10) equations one can obtain:

$$\Delta h_l = \frac{4 f_0^2}{3l}\left[\frac{(1 + 5\gamma/4 + 7\gamma^2/16)}{(1 + \gamma + 5\gamma^2/16)} - 1\right]. \tag{27}$$

Analysing this relation one can find that the horizontal displacements as well as the vertical displacements of the loaded (by complementary asymmetric load part) are directly proportional to loads ratio γ and the cable sag f_0.

Combining the (3), (4) and (10) equations in an analogous way, one can obtain the horizontal displacement of the right cable part for the middle span ($x = l/2$) point:

$$\Delta h_l = \frac{4 f_0^2}{3l}\left[\frac{(1 + 3\gamma/4 + 3\gamma^2/16)}{(1 + \gamma + 5\gamma^2/16)} - 1\right]. \tag{28}$$

Analyzing the expressions (27) and (28) one can find that displacement of left and right cable parts are equal in absolute magnitudes, ie $|\Delta h_l| = |\Delta h_r|$. One must note that the unloaded part cross-sectional displacements of the cable, subjected by asymmetric load p, move left, ie in direction of the loaded cable part. The horizontal suspension cable displacements of the remaining part can be identified via the expression (26), having compared the lengths of the cable parts prior and after loading the cable by asymmetric load.

7. Primary equilibrium form stabilisation of suspension flexible cable

One of the most unfavourable loading cases for a suspension cable that results in maximal kinematic displacements, is the asymmetric loading (Fig 1). The suspension cable primary form stabilisation becomes the main task of the cable-structure design. The maximal kinematic displacements analysis shows these to be directly dependent on the cable sag f_0 and asymmetric and symmetric loads intensities ratio γ. The formulae (18) and (23) show that the satisfaction of the requirement (1) can be achieved by reducing the cable sag f_0 in case of a constant γ. But one must note that the reduction of the cable sag causes large tensile inner forces

$$H_{l1} = \frac{M_l(x = l/2)}{f_0\sqrt{\psi}} = \frac{M_r(x = l/2)}{f_0\sqrt{\psi}} = H_{r1} \qquad (29)$$

that reduce (result finally in) the technical-economical efficiency of the cable structures (leading to enlargement of support equipment mass in order to resist the increase of thrusting forces). The analysis of (18)–(23) formulae yields the kinematic displacements to be zeroes in case of sag absence (or $f_0 = 0$), ie when it serves as a tie. In the latter case the kinematic displacements should be conditioned by elastic deformations only.

The kinematic displacements induced by the fixed asymmetric load p can be stabilised by enlarging the symmetric load size, ie by reducing the ratio γ. But the latter eventual design instrumentality leads to a general load size enlargement, resulting in an enlargement of thrusting forces (29). These forces grow faster as kinematic displacements diminish and proves it to be not an efficient instrumentality. For example, having enlarged the symmetric load q twice, ie when γ is reduced from 2 to 1 magnitude, the displacement decreases approximately by 26 %; when γ is reduced from 4 to 2, the displacement is reduced approximately by 16 %. The relation displacement versus load magnitude q is the non-linear one and it is proved by the above illustration.

One must note that the cable vertical displacements are in direct relationship with horizontal displacements. It is obvious that aiming to reduce vertical displacements, one must reduce the cables possibility to deform horizontally. From formulae (4) and (5) one can find that when reducing the middle span cross-sectional displacement in horizontal direction (eg having introduced horizontal link), the cable left part loaded by complementary asymmetric load will response to loading as independent suspension cable. The span of the latter cable will be $l/2$, and the load p will influence the cable as a symmetric one. No kinematic displacements are induced in such a cable. This method for reducing kinematic displacements is rather efficient as it does not enlarge the thrusting forces in the considered cable. The latter instrumentality is successfully applied for primary stability form stabilisation of bridge structures, when connecting the cable in the middle span with stiffness beam [9, 21] serving as horizontal link. The analogous technical solutions are employed in roof cable structures of buildings [6].

8. Numerical experiments

Numerical simulations have been carried out in order to perform an analysis of asymmetrically loaded suspension cable response in terms of kinematic vertical and horizontal displacements and to fix the errors obtained when applying the known engineering methods to evaluate the above displacements. The 200 m span flexible suspension cable subjected to symmetric and asymmetric loads p (Fig 1) is under investigation. The cable sag f_0, the ratio of symmetric q and asymmetric p loads γ have been varied for displacement analysis.

An analysis of provided simulations proved the kinematic vertical and horizontal displacements to be directly dependent on the cable sag f_0. When increasing the cable sag f_0, vertical displacements increased linearly proportionally. When varying the cable sag f_0 inside the bounds $f_0 = l/4 = 50$ m till $f_0 = l/10 = 20$ m, the left cable part maximal displacement magnitude varied from $\omega_{l,\max} = 1{,}442$ m to $\omega_{l,\max} = 3{,}605$ m in case of the constant (fixed) ratio of loads $\gamma = 1$. It is obvious that aiming to reduce the vertical components of kinematic displacements under the constant load ratio γ, one must reduce the cable sag f_0.

It has been proved that an increase of the load ratio γ results in an increase of the maximal vertical displacements $\omega_{l,\max}$ and $\omega_{r,\max}$ of the left and right cable parts, respectively. One must note that vertical kinematic $\omega_{l,\max}$ and $\omega_{r,\max}$ displacements versus the load ratio γ relation is the non-linear one. When varying γ values in the interval 1–10, an increase of vertical displacements in the first loading stages is relatively large for small γ. But gradually increasing the γ values, one can fix the tendency to reduce vertical displacements (the graph of Fig 2). The latter result shows the cable to be sensitive to the asymmetric load p in the first stages of loading. For instance, under $\gamma = 1$, and for $f_0 = l/10 = 20$ m the left part of vertical displacement is $\omega_{l,\max} = 1{,}442$ m, but for the triple its magnitude, ie for $\gamma = 3$, the magnitude of the above displacement increases approximately by 1,566 and is equal to $\omega_{l,\max} = 2{,}258$ m. Having increased the ratio γ magnitude by six times ($\gamma = 6$), the considered displacement

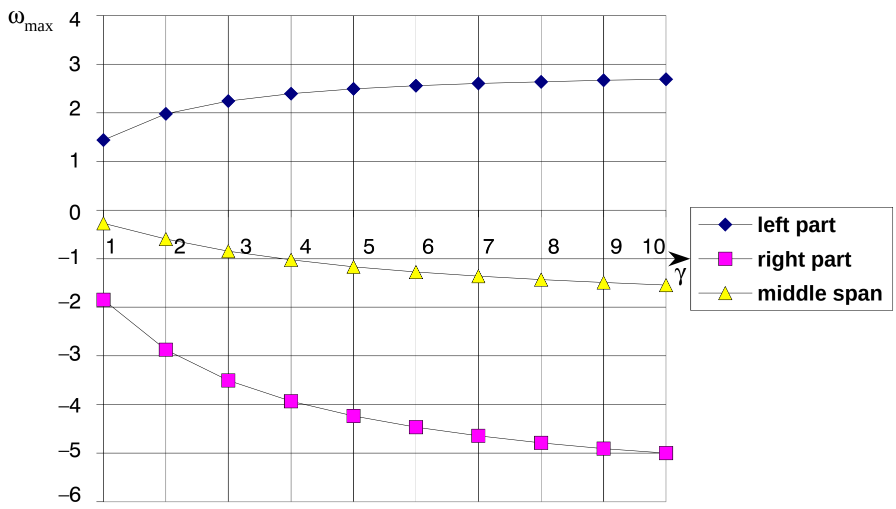

Fig 2. Cable maximal vertical displacements (in m) of left and right parts and middle span versus load ratio γ

reaches the magnitude $\omega_{l,\max} = 2{,}588$ m, ie it increases approximately 1,795 times (the graph of Fig 2).

One must note, that the maximal displacement magnitudes $\omega_{l,\max}$ and $\omega_{r,\max}$ are independent of the cable span value. It has been determined that varying x^{**} in maximal kinematic displacement $\omega_{l,\max}$ zone, its magnitudes change insignificantly. Taking approximately the maximal kinematic displacement $\omega_{l,\max}$ to be located in the cross-section $x^{**} = l/4$, and the maximal displacement $\omega_{r,\max}$ to be located in the cross-section $x^{**} = 3l/4$, their values, calculated by expressions (18) and (23), do not exceed 2 % error. The provided numerical experiment proved the kinematic displacements in absolute values of loaded (left) cable part to be less in comparison with those of the unloaded (right) part (Fig 2). This, looking to be a paradoxical result, is conditioned by negative middle span displacement Δf_k, ie the displacement moves up from its primary position. The analogous distribution of displacements was mentioned briefly in the investigations [17]. Besides, the cable middle span displacement Δf_k, as well as the maximal cable displacements $\omega_{l,\max}$, $\omega_{r,\max}$ increase faster when increasing the loads ratio γ magnitudes. It was found that varying γ from 1 to 10, Δf_k changes from –0,272 m to $-1{,}538$ m under $f_0 = l/10 = 20$ m. Taking $f_0 = l/5 = 40$ m, the γ variation from 1 to 10 results in the Δf_k changes from –0,544 m to –3,076 m. One must note that the application of engineering methods always results in the middle span displacement $\Delta f_k = 0$, for $\gamma \geq 0$.

Having compared the loaded (left) cable part displacement $\omega_{l,\max}$ with the one of the unloaded (right) part $\omega_{r,\max}$, one can find the latter to be larger in percentage (graph of Fig 3). For instance, when $\gamma = 1$, the $\omega_{r,\max}$ is approximately by 28 % larger than the $\omega_{l,\max}$, and when $\gamma = 5$, the $\omega_{r,\max}$ is approximately 70 % larger than the $\omega_{l,\max}$. When $\gamma = 10$, the difference between displacements increase up to 85,7%.

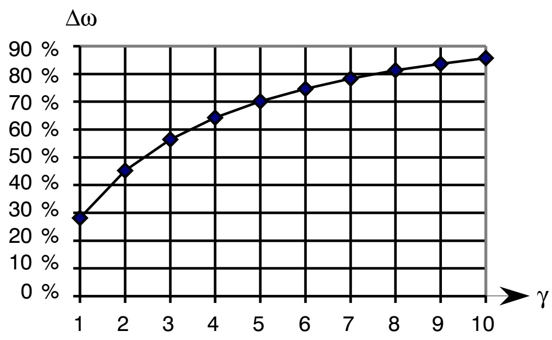

Fig 3. Relative difference graph of vertical displacements corresponding to cable left and right parts

It was numerically found that the maximal curvature change of asymmetrically loaded cable is located in the unloaded right part. In other words, the right part, free of asymmetric complementary load *p*, is relatively more "straightened" then "curved" the left part, subjected by the complementary load *p*. Fig 4 shows that in case of $\gamma = 1$ the curvature change of the unloaded cable part is approximately by 8,73 % larger compared with the one of the loaded part; when $\gamma = 10$, the curvature change difference of both parts increases up to 22,2 %.

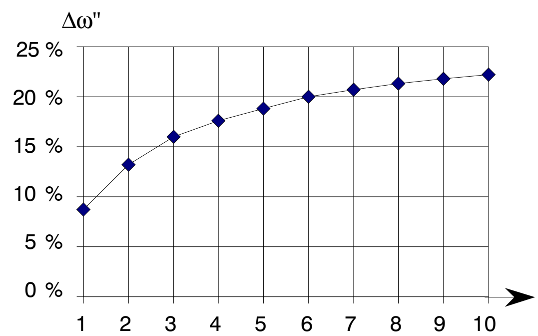

Fig 4. Relative difference graph of cable curvature changes, corresponding to cable loaded and unloaded parts

It has been proved by calculations that widely applied engineering methods, based on superposition principle, result in errors when evaluating vertical kinematic displacements. These errors do not depend on the cable sag f_0, but are sensitive to the load ratio; they increase for the larger γ. When $\gamma=1$, the engineering methods result in the 15,5 % error when calculating $\omega_{l,\max}$; for $\gamma=5$, the error of the latter value increases up to 42 %, and for $\gamma=10$, $\omega_{l,\max}$ the error is of 52 % (Fig 5). The analogous errors are obtained when calculating the vertical displacements of the unloaded right part $\omega_{r,\max}$ (Fig 5).

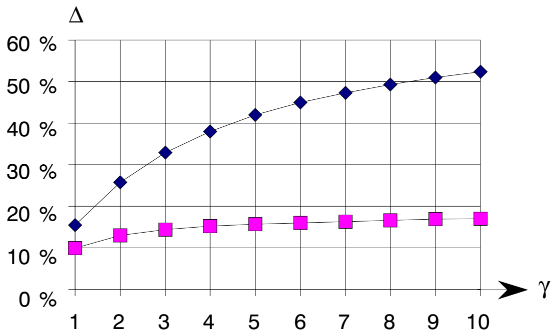

Fig 5. Relative error graph of cable vertical displacements obtained via engineering methods

It is obvious that aiming to satisfy 5 % tolerance constraint, when evaluating vertical displacements of suspension cables, the engineering methods can be employed only when the asymmetric and symmetric loads ratio magnitude satisfy $\gamma\leq1$.

9. Conclusions

The parabolic suspension cable (free of elastic deformations) behaviour versus asymmetric load was investigated. The obtained analytical expressions (15, 21) ensure a more accurate evaluation of kinematic nature vertical displacements of the left and right cable parts. The analysis of the cable behaviour versus loading has shown that an increase of the asymmetric p and symmetric loads q ratio γ results in the increase of the vertical kinematic displacement $\omega_{l,\max}$, $\omega_{r,\max}$ magnitudes. The above displacements relation versus the loads ratio is the non-linear one, the even increase of γ results in the relative reduction of vertical displacements.

It has been determined that the kinematic vertical displacements of the asymmetrically loaded cable part $\omega_{l,\max}$ larger in absolute magnitudes than the ones of the unloaded part $\omega_{r,\max}$. A larger loading ratio γ results in a larger difference between displacements of both parts.

The cable curvature analysis proved that the right cable part is relatively more "straightened" than "curved" the left loaded cable part. The curvature change increases when load ratio γ increases, analogously to kinematic displacements.

The obtained analytic expressions for horizontal displacement evaluation allowed to identify that the middle span cross-section horizontal displacement moves in the loaded cable part direction. This displacement as well as vertical displacements depend on the cable sag and the loading ratio. One can state, the above displacements to be the related ones. Thus, aiming to reduce the vertical displacements, one must reduce the cable sag and increase the symmetric load intensity. Besides, an efficient method to reduce vertical displacements would be the cable horizontal displacements constraining via technical tools.

The presented numerical simulations clearly proved that the errors when estimating kinematic displacements via widely applied engineering methods, based on superposition principle, in case of asymmetric and symmetric loads ratio $\gamma>1$ exceed by 16 %. Therefore the engineering methods are recommended to be applied only in case of $\gamma\leq1$.

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