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OPTIMISATION OF GRILLAGE-TYPE FOUNDATIONS

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1. Introduction

All parts of buildings should be designed and built optimally and thrifty as much as the conditions of safety and comfort allow. In the design of grillage-type foundations this simply means that, firstly, the cross-section of grillage is uniform in all the structure, and secondly, piles supporting the grillage are uniform over all structure, but are placed plausible, not at equal distances from each other. In order to optimally utilise the steel framework of grillage, the bending moments should be uniformly distributed over the structure or, at worst, maximum positive moments should match the minimum ones. Similarly, in order to make the concrete work closely with steel framework of piles, all reactions arising in supports should be as small as possible and uniform.

Thus, the design of economical grillage foundations inevitably is related with optimisation of initial scheme.

The paper deals with the aforementioned problems. We tried to pose the optimisation problems, to define the solution methods, etc, up to the introduction into commercial codes.

2. Statement of problem

The optimisation problem is stated as follows:

Minimise (over feasible shapes) maximum P (over structure and load cases)

with P being the parameter to be optimised.

Two optimisation problems are to be examined: when parameter is maximum bending moment at some points of structure, and maximum vertical reactive force at supports. The feasible shape of structure is defined by the type of certain supports (unmoveable support, spring-support, or support with a given displacement), the given number of different cross-sections and different materials in the structure. During optimisation process new unmoveable supports may appear in the structure, the old supports may merge, however the type of existing supports has to be retained.

The problems should be solved in statics and in linear stage.

Clearly, both problems are highly non-linear. Our choice is to use robust and reliable methods: finite element method for static analysis and linear mathematical programming for optimisation. Thus, the problems have to be solved iteratively and are converted to a sequence of approximately linear problems of an optimal re-design. In each iteration the current shape is changed to a better neighbouring shape. The solution requires three steps:

- finite element analysis
- sensitivity analysis with respect to the co-ordinates of supports
- optimal re-design with linear programming.

Further, the minimum-maximum problem is converted to a pure minimum problem with constraints by treating P_{max} as unknown subject to constraints that P_{max} limits the magnitudes of parameter P everywhere in the structure and for all load cases when design changes Δt_i are performed:

$$P(x)+\sum_i P(x)_{,t_i}\,\Delta t_i - P_{max} \leq 0 \qquad (1)$$

for the total structural space x. The comma here and below means the differentiation.

For computational reasons a length constraint $L=\bar{L}$ is also included:

$$L+\sum_i L_{,t_i}\Delta t_i - \bar{L} = 0. \qquad (2)$$

Several possibilities exist in the choice of design parameters t_i on which the structure shape depends. Our choice is to use the most evident from the engineering point of view design parameters: nodal co-ordinates of all (or a chosen set of) supports.

3. Optimisation technique

With reference to [1, 2] let us shortly describe the optimisation procedures.

Two absolute limits sets (maximum and minimum) on all design co-ordinates status **T** are led up according to existing design restrictions or other considerations. In any case the design variable cannot exceed these limits. For the first solution step, current design variables status $\mathbf{T} = \mathbf{0}$. The absolute limits may differ from one design variable to other, however the maximum absolute move limits must be positive, and the minimum ones negative:

$$\begin{aligned} &\mathbf{T}^{\max} \geq \mathbf{0}, \\ &\mathbf{T}^{\min} \leq \mathbf{0}, \\ &\mathbf{T}^{\min} \leq \mathbf{T} \leq \mathbf{T}^{\max}. \end{aligned} \tag{3}$$

Further, the move limits on the design variables alterations $\mathbf{\Delta T}$ per one iteration are led up, again maximum and minimum. These move limits may vary from one design variable to another and have to be adjusted to the extent of non-linearity of problem so that Simplex' predictions on the future behaviour of the structure do not differ remarkably from finite element solution. In general, move limits should be gradually shrunk as the design approaches the optimum. The accuracy of the approximation is required to be higher when we get close to the optimum because the gains are small and can be swamped by approximation errors. The need to reduce move limits is indicated when the final design of an iteration proves, upon exact analysis, to be inferior to the initial design of that iteration (which is the final design of the previous iteration). Thus,

$$\Delta\mathbf{T}^{\min} \leq \Delta\mathbf{T} \leq \Delta\mathbf{T}^{\max}. \tag{4}$$

Introducing an intermediate always positive variables $\Delta\mathbf{T}^{+}$,

$$\begin{aligned} &\Delta\mathbf{T}^{+} \geq \mathbf{0}, \\ &\Delta\mathbf{T} = \mathbf{T}^{+} + \Delta\mathbf{T}^{\min}. \end{aligned} \tag{5}$$

Hence

$$\Delta\mathbf{T}^{+} \leq \Delta\mathbf{T}^{\max} - \Delta\mathbf{T}^{\min}. \tag{6}$$

Now let us introduce new intermediate variables $\Delta\tilde{\mathbf{T}}$ such that

$$\Delta\mathbf{T}^{+} + \Delta\tilde{\mathbf{T}} = \Delta\mathbf{T}^{\max} - \Delta\mathbf{T}^{\min}. \tag{7}$$

In practical situation when the design variables reach their status limits, the current variable alteration has to be restricted additionally:

$$\text{if} \quad \mathbf{T}^{\max} - \mathbf{T} < \Delta\mathbf{T}^{\max}$$

$$\text{then} \quad \Delta\mathbf{T}^{\max} = \mathbf{T}^{\max} - \mathbf{T}; \tag{8}$$

$$\text{if} \quad \mathbf{T}^{\min} - \mathbf{T} > \Delta\mathbf{T}^{\min}$$

$$\text{then} \quad \Delta\mathbf{T}^{\min} = \mathbf{T}^{\min} - \mathbf{T}, \tag{9}$$

because otherwise the absolute variable changes limits will be exceeded.

After the Simplex' solution the results must be deciphered according to relations rendered below.

1. If resulting design variable $\boldsymbol{i}$ in basis corresponds to a $\Delta\mathbf{T}^{+}$ part of vector of unknowns:

$$\Delta T_i = \Delta T_i^{+} + \Delta T_i^{min}. \tag{10}$$

2. If one corresponds to a $\Delta\tilde{\mathbf{T}}$ part:

$$\Delta T_i = \Delta T_i^{max} - \Delta\tilde{T}_i. \tag{11}$$

3. If both unknowns ΔT_i^{+} and $\Delta\tilde{T}_i$ are presented in basis for one design variable i, the ΔT_i has to be evaluated according to the first or to the second case.

4. If resulting variable corresponds to other parts of the unknowns, then no information for the shape optimisation is obtained, omit this variable.

Now all necessary conditions to the Simplex procedure are satisfied. The problem formulation for mathematical programming is:

Minimise P_{max}

with constraints:

level of P everywhere in the structure $\leq P_{\max}$,

design changes do not exceed move limits, and design status does not exceed absolute limits;

length of model is constant.

Considering only the first derivatives in Taylor's expansion, the first constraints at the nodal points of structure become

$$\mathbf{P} + [P]_{,\mathrm{T}}\, \Delta\mathbf{T} - \mathbf{P}_{\max} \;\le\; \mathbf{0}\,, \tag{12}$$

or avoiding the inequality

$$[I]\,\tilde{\mathbf{P}} - \mathbf{1}\,P_{max} + [P]_{,\mathrm{T}}\, \Delta\mathbf{T}^{+} = - \mathbf{P} - [P]_{,\mathrm{T}}\, \Delta\mathbf{T}^{\min}\,. \tag{13}$$

The second group of constraints in matrix notation for all design variables is:

$$\Delta\mathbf{T}^{+} + \Delta\tilde{\mathbf{T}} = \Delta\mathbf{T}^{\max} - \Delta\mathbf{T}^{\min}\,, \tag{14}$$

while the third one is as follows:

$$L + \sum [L^{e}]_{,\mathrm{T}} \Delta\mathbf{T} = \bar{L} \tag{15}$$

where the sum covers only the active elements, ie including the current design variable as a node of element. In the first iteration $L = \bar{L}$.

4. Finite element. Matrices for sensitivity analysis

Finite element matrices

Simple two-node beam element with 4 d.o.f.'s [3] has been implemented in analysis (Fig 1).

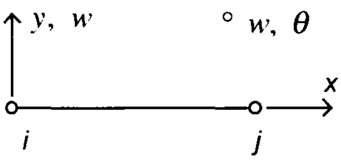

Fig 1. Finite element

Nodal d.o.f.'s of element are:

$$\mathbf{u} = \{ w_i, \theta_i, w_j, \theta_j \}^{T}, \tag{16}$$

w_k and θ_k, $k = i,j$ being deflection and rotation, positive counter-clockwise, accordingly.

The interpolation functions for all d.o.f.'s in Cartesian co-ordinates are as follows:

$$[N] = \begin{Bmatrix} 1 - \dfrac{3x^2}{L^2} + \dfrac{2x^3}{L^3} \\ x - \dfrac{2x^2}{L} + \dfrac{x^3}{L^2} \\ \dfrac{3x^2}{L^2} - \dfrac{2x^3}{L^3} \\ -\dfrac{x^2}{L} + \dfrac{x^3}{L^2} \end{Bmatrix}^{T} \tag{17}$$

with L for length of an element.

Bending moments at nodes, positive when cause the "positive" layers of a finite element experience tension, compile the element stress vector:

$$\boldsymbol{\sigma} = \{ M_i, M_j \}^{T}; \tag{18}$$

flexural rigidity relates them to the deflection:

$$M = -E\,I\, w_{,xx} = E\,I \sum_i N_{i,xx}\, u_i. \tag{19}$$

After the nodal displacements are obtained, the reactive forces are available according to:

$$R_i = \sum_j \mathbf{K}_{ij}\, \mathbf{u}_j. \tag{20}$$

Element loading

Finite element can be loaded by nodal forces and moments, positive counter-clockwise, and by concentrated loads, moments, distributed (of triangular shape) loadings inside the element. Distributed loading (Fig 2) is modified to the statically equivalent loads and moments acting at the end-points of loading:

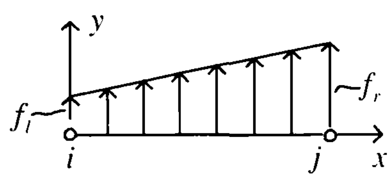

Fig 2. Distributed loading on the element

$$\mathbf{p_q} = \begin{Bmatrix} p_{f_l} \\ p_{m_l} \\ p_{f_r} \\ p_{m_r} \end{Bmatrix} = \frac{l}{20} \begin{Bmatrix} 7f_l + 3f_r \\ l(3f_l + 2f_r)/3 \\ 3f_l + 7f_r \\ -l(2f_l + 3f_r)/3 \end{Bmatrix}. \tag{21}$$

Later on these components as well as all other internal concentrated loads p_f and moments p_m are translated to the nodes of finite element according to general relations of finite element method:

$$\mathbf{p} = [N(x)]\, p_f + [N(x)]_{,x}\, p_m \tag{22}$$

with appropriate co-ordinate x of load application point.

Relations for sensitivity analysis

As seen from (1), the sensitivity (ie derivatives with respect to nodal co-ordinates) of bending moments and reactive forces is the must for optimisation:

$$\mathbf{M}_{,x_k} = -E\,I \sum_i \left(N_{i,xxx_k}\, u_i + N_{i,xx}\, u_{i,x_k} \right), \tag{23}$$

$$\mathbf{R}_{,x_k} = [K]^a_{,x_k}\mathbf{u}^a + [K]^a\mathbf{u}^a_{,x_k}. \qquad (24)$$

with superscript a standing for ensemble.

The derivatives of nodal displacements is obtained by solution of general sensitivity analysis

$$[K]^a\mathbf{u}^a_{,x_k} = \bar{\mathbf{P}}^a, \qquad (25)$$

with pseudo-load vector

$$\bar{\mathbf{P}}^a = \mathbf{P}^a_{,x_k} - [K]^a_{,x_k}\mathbf{u}^a. \qquad (26)$$

The procedure for derivative of element stiffness matrix from which matrix of ensemble $[K]^a{}_{,x_k}$ is composed, is as follows: replace L with x_j-x_i, detect whether k is ith or jth node of an element, and obtain $[K]_{,x_i}$ or $[K]_{,x_j}$, respectively. Thus, only the element possessing node k renders non-zero stiffness derivatives.

Similar procedures are valid for derivatives of forces and reactions.

All matrix expressions are presented in Appendix.

5. Program

The finite element computational procedure, sensitivity analysis and optimal re-design via linear programming form the programs kernel which is supplemented with pre- and post-processing capabilities.

The initial finite element mesh is prepared automatically, leading up nodes at support places, jumps of material and cross-sections properties, etc. Rather dense finite element mesh is necessary, primarily due to the only evaluation procedure of bending moments at mesh nodes. The moment at certain node is calculated via arithmetic mean of bending moments obtained from neighbouring finite elements, and this makes the moments derivatives more sensitive to the finite element length than, for instance, derivatives of reactive forces.

The pre-processor allows the "master nodes", ie, nodes co-ordinates of which are design variables in optimisation procedure, to move over structure freely. The met simple nodes are jumped over when master node approaches these nodes to a specified by program distance. Of course, this causes small numerical disturbances in sensitivity analysis. When two or more master nodes meet, the following procedures govern:

- support-support: one of supports is deleted if resulting reactive force is less than the given allowable reaction. If resulting reaction is not acceptable, the supports are fixed at minimum allowed distance between, and removed from master nodes.
- support– spring-support: spring-support is removed from master nodes
- spring-support – spring-support: one of supports is deleted by adding spring stiffnesses of both supports.

Resuming, the optimisation is an intellectual process. It is impossible to write "one button click" program which automatically will render optimal solution. Linear programming may lead to a local minimum, therefore the problem solution from different starting positions is recommended. Similar situation is shown in examples below. Also, varying iteration move limits may help.

6. Optimisation of bending moments

A number of numerical examples demonstrate the capabilities of proposed model. The first examples are symmetric to be able to compare the obtained results with in advance known optimal shapes of beams. From the engineering point of view it is important to minimise not the maximum, but maximum in absolute value bending moment, therefore special procedures were introduced into codes to incorporate this approach.

Example 1. Beam on 4 fixed supports loaded with uniformly distributed loading. Let us start from in advance known non-optimal layout of supports (Fig 3).

Three supports are chosen to be master nodes. The move limits of all master nodes during the whole optimisation process were ± 0.1; for those magnitudes the finite element solutions correspond sufficiently to the Simplex predictions.

All remaining nodes are placed at equal distances. The starting magnitudes of bending moments together with their locations are (Fig 4 a):

M_{max} = 47.93 at node 15,

M_{min} = -72.18 at node 5.

Actually, M_{min} was minimized. After 10 iterations support 3 reaches support 1 and is removed from master nodes. Optimisation of remaining three-supports beam ends after 30 iterations (Fig 4 b) with

M_{max} = 17.58 at node 5,

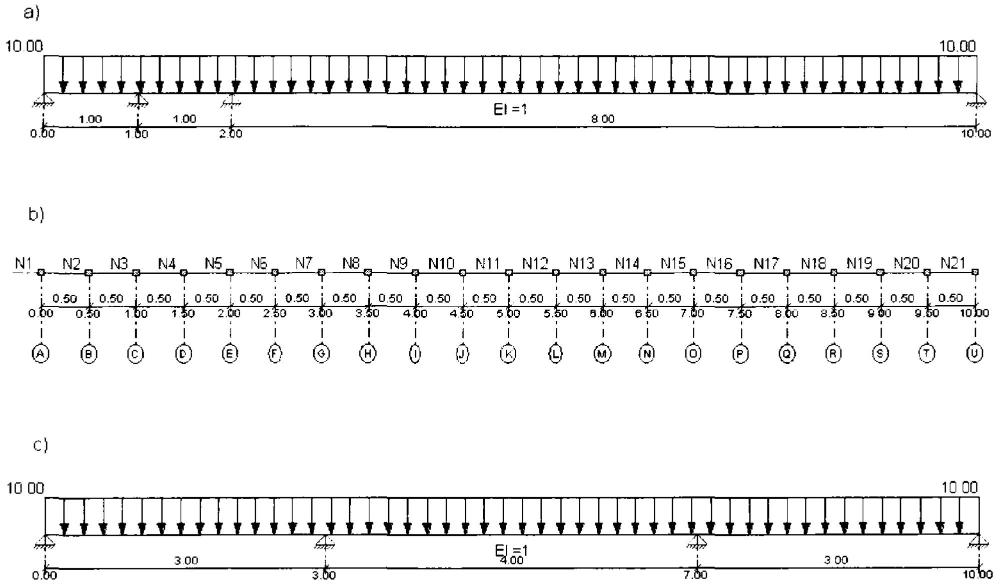

Fig 3. Beam under uniformly distributed loading: (a) initial scheme I, (b) finite element mesh, (c) initial scheme II

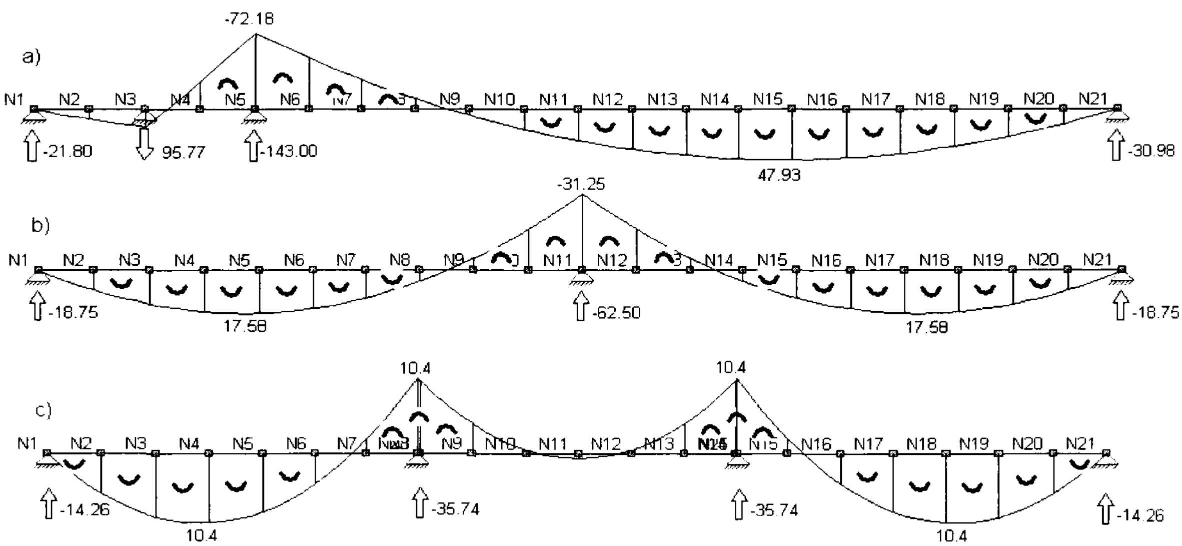

Fig 4. Optimisation results of beam under uniformly distributed loading: (a) bending moments and support reactions distribution for initial scheme I before optimisation, (b) bending moments and support reactions distribution for initial scheme I after optimisation, (c) bending moments and support reactions distribution for initial scheme II after optimisation

M_{min} = -31.25 at node 11,

and final co-ordinates of supports 0.0, 5.0, 10.0.

Now, let us take solution from other starting positions (Fig 3 c). Final results are achieved in 15 iterations (Fig 4 c):

M_{max} = 10.4 at nodes 4, 18;

M_{min} = -10.4 at nodes 8, 14

by co-ordinates of supports 0.00, 3.48, 6.52, and 10.00.

Example 2. Beam with fixed and spring supports, and supports with prescribed non-zero displacements (Fig 5 a). Finite element mesh is the same as in Fig 3 c Master nodes are all support-nodes including the node with prescribed translational stiffness plus the first node, all with move limits ± 0.1. Initial solution is (Fig 5 b):

M_{max} = 6.62 at node 11,

M_{min} = -8.92 at node 13.

Due to the high gradients the problem converges fast and final results after 14 iterations are:

M_{max} = 2.51 at node 6;

M_{min} = -2.13 at node 4

by co-ordinates of supports 1.10, 3.10, 5.93, and 8.80.

7. Optimisation of vertical support reactions

Let us solve the same problems but minimising vertical support reactions, again maximum in absolute value.

Example 1 (Fig 3 a, b). The problem solution with previous move limits leads to a deadlock. Starting with

R_{max} = 95.77 at node 3,

R_{min} = -143.0 at node 5,

after the 7th iteration the maximum in absolute value magnitude of reaction (-88.4 at node 6) grows to a – 102.7 at node 2. However, even a double-reduction of negative move limit for the 3rd master node allows us to solve the problem as expected: after the 20th iteration support (initial node 3) reaches left support and is deleted, then after 31 iterations

R_{max} = -18.75 at nodes 1, 20;

R_{min} = -62.50 at node 10

by co-ordinates of supports 0.0, 5.0, 10.0 (Fig 4).

The initial scheme II (Fig 3 c) leads exactly to the same results.

Example 2. (Fig 5 a). This problem converges per 58 iterations, yielding 3 supports (Fig 5 c), from

R_{max} = 19.6 at node 11,

R_{min} = 4.46 at node 17

to a

R_{max} = 9.87 at node 17,

R_{min} = -10.01 at node 7

by co-ordinates of supports 0.00, 3.20, 7.90.

8. Common optimisation of bending moments and supports

The most desirable situation for engineering practice is to have in a grillage as small as possible vertical reactive forces and bending moments together. However, the solutions in previous chapters indicate clearly, that these aims are not compatible. Some engineering solutions are needed for joint optimisation of reactions and moments. They are as follows: the program starts optimisation of reactions and proceeds with it until allowable magnitude of reaction is obtained; then shifts to the optimisation of bending moments. Backward shift occurs when improper reaction emerges, etc.

Example 1 (Fig 3 a, b). Allowable vertical reaction is set to a 100. Move limits and starting magnitudes of bending moments and reactions are given in previous chapters. The solution begins with optimisation of reactions. After the 6th iteration (R_{min} =-93.5 is achieved) process shifts to a optimisation of moments. The optimisation finishes in 57 iterations with

M_{max} = 12.00,

M_{min} = -12.00,

R_{max} = -13.68,

R_{min} = -37.4

by co-ordinates of supports 0.00, 3.65, 6.67, 10.00.

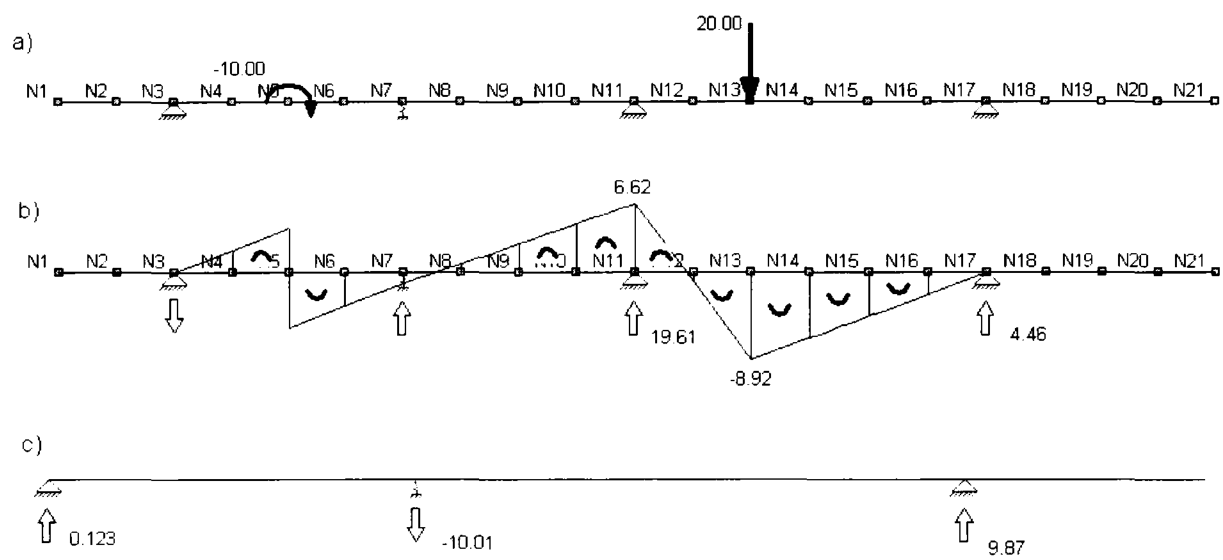

Fig 5. Beam with different kind of supports: (a) initial scheme, (b) bending moments distribution after optimisation, (c) support reactions distribution after optimisation

Example 2. (Fig 5 a). $R_{allowable} = 20$, move limits are kept the same. Solution shifts from optimisation of reactions to an optimisation of reactions after the 7th iteration (R_{max}=13.87). Final solution is obtained after 60 iterations:

$M_{max} = 3.13$,

$M_{min} = -7.74$,

$R_{max} = 12.42$,

$R_{min} = 2.73$

by co-ordinates of supports 0.00, 2.10, 5.30, 7.10.

9. Concluding remarks

The mathematical models for optimisation of grillage-type foundations are presented. Minimising of maximum in absolute value vertical reactive force, bending moment, and reaction-bending moment together is sought. Solutions of a number of problems demonstrate the validity of proposed algorithms. New investigations of merit functions are needed for the case of joint optimisation of reactions and moments.

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Appendix. Matrix relations for sensitivity analysis

$$[K] = \frac{E\ I}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ & 4L^2 & -6L & 2L^2 \\ & & 12 & -6L \\ & sym. & & 4L^2 \end{bmatrix}$$

$$[K]_{,x_i} = \frac{E\ I}{L^4}\begin{bmatrix} 36 & 12L & -36 & 12L \\ & 4L^2 & -12L & 2L^2 \\ & & 36 & -12L \\ & sym. & & 4L^2 \end{bmatrix}$$

$$[K]_{,x_j} = -[K]_{,x_i}$$

$$\mathbf{p}_{,x_i} = \frac{1}{L^2}\begin{Bmatrix} 6p_{m_i} \\ -L^2 p_{f_i} + 4Lp_{m_i} \\ -6p_{m_i} \\ 2Lp_{m_i} \end{Bmatrix}$$

$$\mathbf{p}_{,x_j} = \frac{1}{L^2}\begin{Bmatrix} -6p_{m_j} \\ -2Lp_{m_j} \\ 6p_{m_j} \\ -L^2 p_{f_j} - 4Lp_{m_j} \end{Bmatrix}$$

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PAMATŲ SIJYNO OPTIMIZAVIMAS

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Santrauka

Suformuluoti pamatų sijyno optimizavimo uždavinių matematiniai modeliai, pasiūlyti jų sprendimo algoritmai. Išspręsti trys uždaviniai: lenkimo momentų, atraminių reakcijų minimizavimo ir bendras reakcijų ir lenkimo momentų minimizavimo uždavinys. Statikos uždavinys sprendžiamas baigtinių elementų metodu. Panaudotas paprastas dviejų mazgų lenkiamo strypo elementas su dviem laisvumo laipsniais mazge. Jautrumo atramų postūmiams analizė atlikta analitiškai. Optimizavimui taikytos tiesinio matematinio programavimo procedūros. Kadangi optimizavimo uždavinys yra aiškiai netiesinis, programoje įdiegtas iteracinis interaktyvus patikslinimo procesas, leidžiantis keisti atramų postūmius taip, kad optimizavimo procedūros numatymai reikiamu tikslumu atitiktų baigtinių elementų metodo sprendinius. Pirmųjų dviejų tipų uždaviniai suformuluoti griežtai, o bendrajam momentų/reakcijų optimizavimo uždaviniui taikytos tam tikros inžinerinės prielaidos. Pateikti visų trijų optimizavimo uždavinių sprendiniai įvairioms pamatų sijoms.

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