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A NEW HYBRID METHOD FOR SIZE AND TOPOLOGY OPTIMIZATION OF TRUSS STRUCTURES USING MODIFIED ALGA AND QPGA

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Abstract. Modified Augmented Lagrangian Genetic Algorithm (ALGA) and Quadratic Penalty Function Genetic Algorithm (QPGA) optimization methods are proposed to obtain truss structures with minimum structural weight using both continuous and discrete design variables. To achieve robust solutions, Compressed Sparse Row (CSR) with reordering of Cholesky factorization and Moore Penrose Pseudoinverse are used in case of non-singular and singular stiffness matrix, respectively. The efficiency of the proposed nonlinear optimization methods is demonstrated on several practical examples. The results obtained from the Pratt truss bridge show that the optimum design solution using discrete parameters is 21% lighter than the traditional design with uniform cross sections. Similarly, the results obtained from the 57-bar planar tower truss indicate that the proposed design method using continuous and discrete design parameters can be up to 29% and 9% lighter than traditional design solutions, respectively. Through sensitivity analysis, it is shown that the proposed methodology is robust and leads to significant improvements in convergence rates, which should prove useful in large-scale applications.

Keywords: structural optimization, finite element analysis, augmented Lagrangian, quadratic penalty function, hybrid genetics algorithm.

Introduction

Structural optimization techniques are effective tools that can be used to obtain lightweight, low-cost and high performance structures. Optimum design of truss structures has been widely studied by many researchers as they represent a common and complex category of engineering structures. The size and topology optimization of truss structures is a mixed variable optimization problem, which deals simultaneously with discrete and continuous design variables (Šilih *et al.* 2010). Such problems are usually non-convex by nature and, therefore, must be solved by appropriate optimization methods. Topology optimization studies are usually based on the assumption of an initial ground structure that contains all possible joints and members. While most of conventional mathematical optimization methods are suited and developed for continuous design variables (e.g. Hajirasouliha *et al.* 2011), in practice many structural design variables are chosen based on discrete values due to manufacturing constraints. Zhang *et al.* (2013) presented a comprehensive study on discrete optimization using generalized shape function-based parameterization. Genetic algorithms (GAs) have been recognised as one of the most powerful stochastic optimization methods for optimum design of truss structures, where the search space involves both discrete and continuous domains (Adeli, Sarma 2006). GA, in general, represents adaptive search techniques that simulate natural inheritance by adopting appropriate models based on genetics and natural selection. Rahami *et al.* (2008) applied a combination of force method, energy concept and GA for optimum design of different types of truss structures. They included the material and geometric nonlinearity, which are essentially important in the seismic deign of structures. Hasançebi (2007) used a different method for optimization of truss bridges by combining various variable-wise versions of adaptive evolution strategies under a common optimization routine. They carried out size and shape optimizations by using discrete and continuous evolution strategies, respectively. Ant System algorithm is another method that is used by Luh and Lin (2008) to find optimal truss structures for achieving minimum weight under stress, deflection, and

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kinematic stability constraints. The results of their study indicated that multiple truss topologies with almost equal overall weight can be found concurrently as the number of members in the ground structure increases. Dede *et al.* (2011) combined GA with value and binary encoding for continuous and discrete optimization of trusses to minimize structural weight based on stress and displacement constraints. They showed that the value encoding method requires less computer memory and computational time to achieve optimum solutions.

This paper aims to develop an efficient hybrid GA method for size and topology optimization of truss structures using both continuous and discrete design variables. To achieve a good convergence, binary, integer and floating-point encoding is utilized. Hybrid GA is introduced to overcome inequality and equality constraints applied to the structure. Augmented Lagrangian Genetic Algorithm (ALGA) and Quadratic Penalty Function Genetic Algorithm (QPGA) methods are proposed for continuous and discrete non-linear optimization of truss structures, respectively. For size optimization, the cross-sectional areas of the members are selected as design variables, while topology optimization is associated with connectivity of the elements between the nodes. The efficiency of the proposed methods to obtain reliable optimum solutions is investigated through sensitivity analysis.

1. Optimization methodology

1.1. Objective function

There are several criteria for optimum design of truss structures including weight, cost, displacements, maximum stresses, bucking strength, vibration frequencies, or any combination of these parameters. In this study, the objective function is to minimize the weight of the structure, as shown in Eqn (1):

$$Minimize: f(A)^{\mathrm{p}} = \sum_{j=1}^{N} \rho_j L_j A_j \, , \qquad (1)$$

where N is the number of truss structural members, ρ is the density of the material, p is the optimum possibility for the weight function, L is the length of each truss member, and A is the cross-sectional area of the members.

During optimization process, A_i can either be continuous, chosen to be random number within a set region or can be discrete values extracting from cross-section types available in the market. The Augmented Lagrangian method is applied for solving the constrained optimization problem. To allow non-linear constraints, Karush-Kuhn-Tucker (KKT) conditions are utilized. Therefore, to minimise the objective function, the following KKT equation should be satisfied:

$$\nabla f\left(A^*\right) + \sum_{i=1}^{N_{cg}} \lambda_i . \nabla g_i\left(A^*\right) + \sum_{i=1}^{N_{ch}} \gamma_i . \nabla h_i\left(A^*\right) = 0 \, ;$$

$$\text{Subject to: } \gamma_i . h_i\left(A^*\right) = 0,\ i = \left[1 : N_{ch}\right] \ \&$$

$$\lambda_i \geq 0,\ i = \left[1 : N_{cg}\right], \qquad (2)$$

where λ_i is Lagrange multipliers, $f(A)$ is the objective function, $g_i(A)$ and $h_i(A)$ are inequality and equality constraint functions, respectively. Also, N_{cg} and N_{ch} denote the numbers of inequality and equality constraints, respectively.

1.2. Constraint handling

In this study, the constraints were basic nodes, structural stability, member stress, nodal displacement, and buckling strength. Structural stability of the truss was also examined for external and kinematic stability. Using GA concepts, topological instability in each chromosome was determined before structural analysis. The penalty function was allocated to the related chromosome in the unstable truss structure. The kinematic stability of structure should have a symmetric and positive definite stiffness matrix in Cholesky method. Therefore, the Cholesky approach for the stiffness matrix [*K*] was employed for the internal instability checks during the optimization process. In connection with member stress, the stress resulting from design load combinations should be within allowable limits, according to the materials used. In this study, a number of penalty functions were determined with regard to allowable tension and compressive stress of the truss members. If any one of the constraints is not satisfied, a penalty function is assigned to the related chromosome by using Eqn (3):

$$P_{1i}\left(A; X = Ten, Com\right) = \max\left(\left|\frac{\sigma_{i(X)}}{\sigma_i^a(x)} - 1\right|, 0\right), \qquad (3)$$

where the i^{th} member can be under tension or compression, $P_{1i}(A, X)$ is the penalty function value for the stress, σ_i and σ_i^a are the member stress and allowable stress, respectively. In this study, FE analysis was used to calculate the member stress and nodal deflection of the truss structure in the optimization process. Similar to the stress constraints, if any one of the displacement constraints is not satisfied, a penalty function for the vertical displacement is assigned to the related chromosome by using Eqn (4):

$$P_{2i}(A) = \max\left(\left|\frac{\Delta_i}{\Delta_i^a} - 1\right|, 0\right); i = 1, \ldots, N_{joint} \, , \qquad (4)$$

where $P_{2i}(A)$ is the penalty value of the active nodal displacement, Δ_i is the displacement in the direction of the degree of freedom, and Δ_i^a is the allowable displacement in the direction of the degree of freedom.

In general, the failure of a truss structure could be due to failure of a structural component, material failure or structural instability. In this study, tubular hollow sections were used for all truss members with outer width (*d*), inner width (*c*) and sectional thickness (*t*). The buckling strength of each member was calculated based on the ratio of inner width (*c*) to sectional thickness (*t*) according to Eurocode 3 (2010). Cross sections were placed into one of four behaviour classes in Eurocode 3 (2010) defined by $\frac{d}{t \times \varepsilon^2}$, in which ε is $\sqrt{235/f_y}$. To avoid buckling in the compressive members, the following criterion should be satisfied:

$$x = \frac{N_{ED}}{N_{b.Rd}} \leq 1.0, \tag{5}$$

where N_{ED} is the design axial load and $N_{b.Rd}$ is the member buckling resistance determined based on the following equation:

$$N_{b,Rd} = \{\frac{\chi A f_y}{\gamma_{M1}}, \frac{\chi A_{eff} f_y}{\gamma_{M1}}\}, \tag{6}$$

where the first term corresponds to Classes 1, 2 and 3 of the Eurocode 3 (2010) cross sections, while the second term is for Class 4 sections. Also, χ is the reduction factor of the relevant buckling mode, *A* is the gross area, A_{eff} is the reduced effective area, and γ_{M1} is the partial safety factor for buckling resistance calculations. For members under compression, the value of χ should be determined for the appropriate non-dimensional slenderness ratio $\bar{\lambda}$ from the relevant buckling curve, according to Eqn (7):

$$\chi = \frac{1}{\varphi + \sqrt{\varphi^2 - \bar{\lambda}^2}};\ \chi \leq 1.0, \tag{7}$$

where

$$\varphi = 0.51 + \alpha\left(\bar{\lambda} - 0.2\right)\bar{\lambda}^2 \text{ and } \bar{\lambda} = \left\{\sqrt{\frac{\mathrm{Af_y}}{\mathrm{N_{cr}}}}, \sqrt{\frac{\mathrm{A_{eff} f_y}}{\mathrm{N_{cr}}}}\right\},$$

where the first term is applied for Class 1, 2 or 3 and the second term is applied for Class 4 cross sections. $\mathrm{N_{cr}}$ is the elastic critical bucking load based on the gross cross-sectional properties, $\mathrm{N_{cr}} = \frac{\pi^2 \mathrm{EI}}{\mathrm{L}^2}$. The imperfection factor, α, depends on the cross-section type. In this study, α was considered to be 0.49. Based on Eurocode 3 (2010), for $\bar{\lambda}$ less than or equal to 0.2, buckling effects can be ignored.

1.3. Augmented Lagrangian Genetic Algorithm (ALGA) for continuous optimization approach

ALGA was used for solving nonlinear optimization of truss structures with nonlinear constraints. This method helps to avoid conducting extensive numerical calculations to find the appropriate value for the penalty function coefficient. In this way, each constraint is separately allocated to its own adjusted penalty function coefficient. The advantage of using ALGA in comparison with QPGA is to include a set of Lagrange Multipliers, instead of a single coefficient penalty function. The fitness function and nonlinear constraint functions are combined by using the Lagrangian and the penalty parameters for a sequence of sub-problems. Subsequently, each sub-problem is solved by using genetic algorithm. The algorithm starts by setting an initial value for the penalty parameter (i.e. initial penalty). The sub-problem formulation is defined by Eqn (8):

$$\varphi(A,\lambda,s,\xi) = f\left(A\right)^p - \sum_{i=1}^{m} \lambda_i s_i \log\left(s_i - g_i\left(A\right)\right) + \sum_{i=m+1}^{mt} \lambda_i g_i\left(A\right) + \frac{\xi}{2} \sum_{i=m+1}^{mt} g_i\left(A\right)^2, \tag{8}$$

where *m* and *mt* are the number of nonlinear inequality constraints and the total number of nonlinear constraints, respectively. The components λ_i of the vector λ are known as Lagrange multiplier estimates, the elements s_i of the vector s are nonnegative shifts, and ξ is the positive penalty parameter.

1.4. Quadratic Penalty Function Genetic Algorithm (QPGA) for discrete optimization approach

The constrained optimization problems can be converted into unconstrained problems using the QPGA method. This approach requires the selection of a penalty function coefficient. In this study, the penalty function is defined by Eqn (9):

$$\bar{\varphi}\left(A,\gamma\right) = L_f\, f\left(A\right)^p + \gamma_1 \left\{ \sum_{i}^{N} \left[\max\left(\frac{|\sigma_i|}{|\sigma_i^a|} - 1, 0 \right) \right]^2 + \gamma_2 \sum_{i}^{M} \left[\max\left(\frac{|\Delta_i|}{|\Delta_i^a|} - 1, 0 \right) \right]^2 \right\}, \tag{9}$$

where γ_1, γ_2 are the penalty function coefficients, L_f is the factor for normalizing the objective function, *N* is the number of design variables, *M* is the number of degrees of freedom, σ_i is the maximum stress in the i^{th} member, and δ_i is the displacement of each node in vertical direction. Since discrete optimization indicates greater sensitivity on nodal displacements, Eqn (10) is introduced to obtain fast convergence:

$$\varphi\left(A,\gamma\right) = 10^n \times \left\{\max\left(|\delta|\right) - \delta_i^a\right\} + 10^k \times \max\ \{f\left(A\right)^p_{\max} - \left(W\right)^p, 0\} + 10^{-m}\bar{\varphi}\left(A,\gamma\right), \tag{10}$$

where *n*, *k* and *m* are penalty coefficients considered to be 8, 2 and 5, respectively.

1.5. Procedure for obtaining the optimum solution

Figure 1 shows the flowchart of the proposed optimization methods. In the adopted GA optimization approach, the population size, maximum number of generations, stall generation and stall time limit generation were 40,

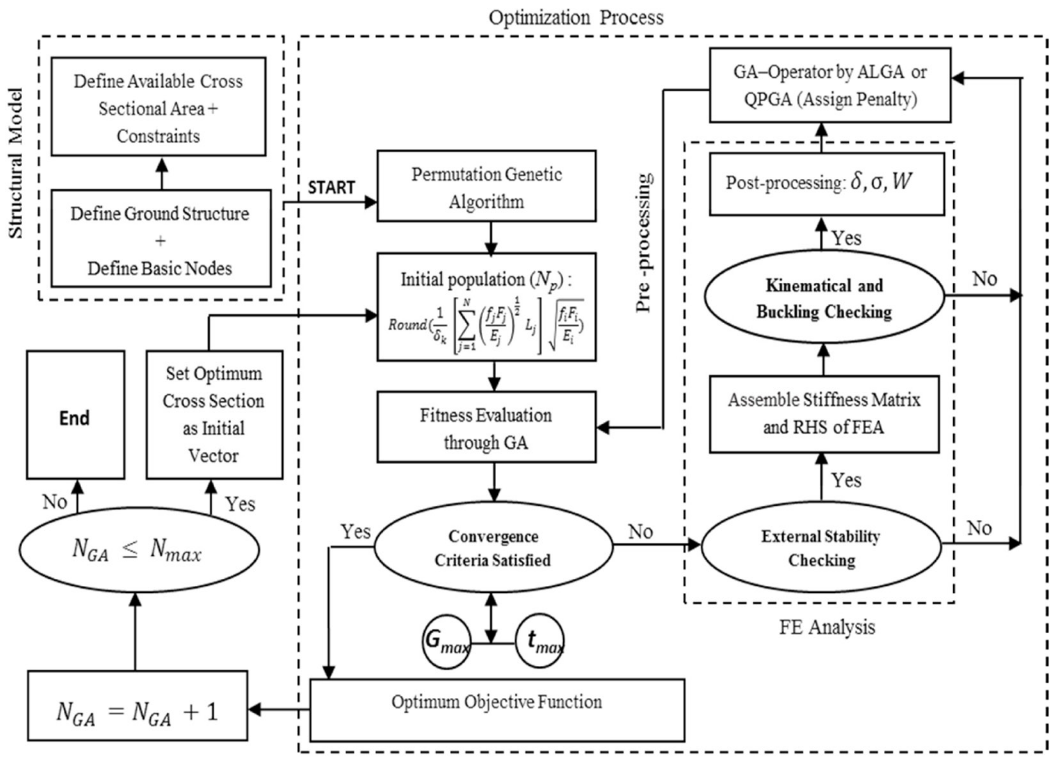


Fig. 1. The flowchart of the proposed optimization process

200, 50, and 200, respectively. Additionally, the penalty coefficient and the tolerance of the nonlinear constraint violation in the optimization process were considered as 100 and 1e^{-6}, respectively. The method starts from an optimal population represented by the cross-sectional areas, that minimises the objective function to a required accuracy. Then, the solution is filtered into FE modeling to be kinematically stable, i.e., until Gruber's criterion is satisfied and the existence of the basic nodes can be checked out by the model. Subsequentely, different configurations are created by GA procedure. The kinematic stability and buckling of the structure is then verified by the Cholesky factorization within the global stiffness matrix. The process reiterates by assigning an initial population based on the optimal solution founded in the first step of optimization. The effeciency of the successive model is then improved by updating the initial population using the last optimum encountered solutions. The maximum numer of iterations, N_{max}, was set to be 10. Since the unconstrained stiffness matrix is a sparse banded, symmetric and positive definite matrix, it is possible to reduce arithmetic operations by using Cholesky factorization. However, density of the Cholesky factorization affects the computational time and cost. Furthermore, for sparse matrices, iterative methods need to be considered to improve the efficiency of the optimisation process. As a result, the full stiffness matrix $O(N^3)$ can be reduced to $O(N)$, where N is the size of the matrix. In this way, the computational time can be reduced more than 5 times (Jhurani, Demkowicz 2012). In this paper, Compressed Sparse Row (CSR) format and reordering of Cholesky factorization of stiffness matrix by symmetric approximate minimum degree permutation is used. Furthermore, in the case of the singular finite element stiffness matrix, Moore Penrose Pseudoinverse though developed Singular Value Decomposition (SVD) is employed.

2. Case study examples

2.1. Benchmark case study

The performance of the proposed optimization method is tested for the benchmark 10-bar cantilever truss shown in Figure 2. The results are compared with several other research studies using both discrete and continuous design variables.

Continuous and discrete variables were represented by float and permutation coding, respectively. The objective

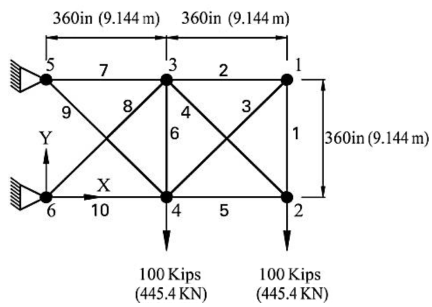


Fig. 2. Geometry of the benchmark10-bar truss

Table 1. Comparison of the continuous size optimization results with other references (Max *y*-displacement: 17.78 cm)

Area	Auer (2006)	Romero *et al.* (2004)	Burton (2004)	Haftka, Gurdal (1982)	de Souza and Fonseca (2008)	Present Work (1)	Present Work (2)
A1	0.645	0.645	0.645	0.645	0.645	0.645	0.645
A2	0.645	0.645	0.645	0.645	0.645	1.419	1.587
A3	0.751	0.645	0.903	0.903	0.839	1.710	2.303
A4	35.878	35.928	23.742	23.742	24.903	36.161	38.071
A5	25.370	25.405	52.258	25.161	25.161	31.632	27.716
A6	0.645	0.645	0.645	0.645	0.645	0.645	0.645
A7	51.177	51.212	50.968	50.968	50.968	53.503	55.722
A8	35.878	35.928	35.548	35.548	35.613	36.581	37.993
A9	37.114	37.063	37.419	37.419	37.290	31.110	28.077
A10	52.050	52.013	25.161	52.258	52.193	48.574	48.852
Weight (Kg)	722.765	722.647	679.251	679.251	682.783	723.118	723.125
Max Disp. (cm)	18.2880	18.2880	22.0675	20.5740	20.2743	17.5971	17.3965
Max Stress(MPa)	172.368	172.372	352.463	258.372	246.762	215.582	224.459

function is to minimize total mass or equivalently, the cross sectional area of the truss members subjected to design constraints. For discrete design variables, the cross-sectional areas were selected from the following 32 predefined sections: {10.452, 11.613, 15.355, 16.903, 18.581, 19.935, 20.193, 21.806, 23.419, 24.774, 24.968, 26.968, 28.968, 30.968, 32.064, 33.032, 37.032, 46.580, 51.419, 74.193, 87.097, 89.677, 91.613, 100, 103.226, 121.290, 128.387, 141.935, 147.742, 170.967, 193.548, 216.129} cm^2. For continuous optimization, the lower and upper bounds of the cross-section areas varies between 0.645 and 64.516 cm^2.

2.1.1. Size optimization using continuous cross-sectional areas

In this section, size optimization was conducted to determine the optimal cross-sectional area of each member with a continuous float value. In this study, mild steel was used in the truss elements. To take into account the nonlinear constraints applied to the structure, ALGA optimization method is utilized. Table 1 shows the comparison of size optimization results with those of other research studies. The results of this study for minimum displacement and minimum weight are shown in the present work (1) and present work (2) columns, respectively. It should be noted that some of the studies (e.g. Romero *et al.* 2004) did not consider displacement constraints in the optimization process and, therefore, obtained structures with less structural weight.

2.1.2. Size optimization using discrete cross-sectional areas

By considering discrete values as design variables, the cross-sectional areas are to be chosen from a set of discrete values of commercially available sizes presented in Section 2.1. Table 2 shows the comparison of size optimization results in this study with results from literature. Similar to the previous case, the optimum design for minimum displacement and minimum weight in this study are given in the present work (1) and present work (2) columns, respectively. As expected, Table 2 shows that according to the release of the displacement constraints, lighter structures may be achieved. However, the optimum solution in this case may not satisfy the maximum displacement limits (see Wu and Chow (1995) results in Table 2).

2.1.3. Size and topology optimization using discrete cross-sectional areas

The objective of the topology optimization in this study is to obtain a truss structure with optimum layout that satisfies all design constraints using minimum structural weight. The topology optimization process starts with an initial ground structure that contains all possible joints and members, followed by eliminating inefficient members, taking into account the instability effect. Variables involved in the optimization process can be 1 or 0 values, representing the presence or absence of the element. The truss structures with fewer structural members are encouraged by assigning smaller penalty function values, while higher penalty values are used for truss structures with larger number of connectivity. The penalty constants in this study were assigned as -10^3 and 10^3 for eliminating or adding a member, respectively. Table 3 compares results obtained by the proposed algorithm with other references. The optimal design for the minimum displacement and weight is shown in present work (1) and present work (2) columns, respectively.

2.1.4. Sensitivity analysis of 10-bar truss structure

In the following, the response of the optimal area distribution to an adjustment of constraints (perturbation) for

Table 2. Comparison of the discrete sizing optimization results with other references (Max y-displacement: 5.08 cm)

Area	Wu and Chow (1995)	Rajeev and Krishnamoorty (1992)	Rahami *et al.* (2008)	Li *et al.* (2009)	de Souza and Fonseca (2008)	Present Work (1)	Present Work (2)
A1	10.452	10.452	11.613	10.452	10.452	18.581	11.613
A2	10.452	10.452	10.452	11.613	10.452	24.774	23.419
A3	15.355	16.903	11.613	19.935	10.452	24.968	21.806
A4	121.290	128.387	141.935	141.935	141.935	121.290	128.387
A5	91.613	100.000	87.097	100.000	100.000	87.097	87.097
A6	11.613	10.452	10.452	10.452	10.452	10.452	16.903
A7	170.967	216.129	193.548	193.548	193.548	193.548	216.129
A8	103.226	128.387	121.290	121.290	141.935	121.290	121.290
A9	33.032	91.613	74.193	74.193	51.419	87.097	74.193
A10	103.226	141.935	193.548	170.967	170.967	147.742	170.967
Weight (Kg)	1985.009	2546.393	2531.842	2537.140	2492.632	2489.882	2568.985
Max Disp. (cm)	6.6462	5.0820	5.0762	5.0665	5.0889	5.2730	5.0825

Table 3. Comparison of the discrete size and topology optimization results with other references (Max y-displacement: 5.207 cm)

Area	Rajan (1995)	Tang *et al.* (2005)	Rahami *et al.* (2008)	Present Work (1)	Present Work (2)
A1	0	0	0	0	0
A2	0	0	0	0	0
A3	0	0	0	0	0
A4	141.935	121.290	141.935	103.226	121.290
A5	100.000	91.613	100.000	91.613	100.000
A6	0	0	0	0	0
A7	193.548	193.548	193.548	193.548	170.967
A8	141.935	128.387	128.387	147.742	121.290
A9	46.581	51.419	46.581	46.581	37.032
A10	128.387	170.967	128.387	147.742	216.129
Weight (Kg)	2250.769	2232.240	2202.280	2160.752	2233.719
Max Disp. (cm)	5.2578	5.2070	5.2034	5.4074	5.2675

the 10-bar truss benchmark is performed. This is achieved by simultaneously increasing the cross-sectional area of members 7 and 10 (highest and lowest cross-sectional area). Figure 3 presents the maximum nodal displacement of the truss structure with respect to the cross-sectional area of the member 7 and 9, respectively. It is shown that the gradient of maximum displacement reaches the minimum value for cross-sectional area equal to 115.48 cm^2 (17.9 in^2) and 71.61 cm^2 (11.1 in^2) in members 7 and 10, respectively. Maximum displacement at this point is 7 cm (2.77 in), which is around 33% above the optimum solutions shown in Table 2. The results also indicate that the maximum displacement is more sensitive to the variations in the cross-sectional area of member 9 compared to member 7. Subsequently, maximum displacement corresponding to member 9 reaches the constant value earlier than member 7.

2.2. Optimum results for 61-bar Pratt truss bridge

As another challenging test example for evaluation of the robust design optimization, the long span 61-bar Pratt truss bridge with 80 m floor deck illustrated in Figure 4a is considered.

All members in the Pratt truss bridge were assumed to be made from the mild steel with E and ρ equal to 210 GPa and 7860 Kg/m^3, respectively. Available cross-sectional areas corresponding with discrete optimization are selected from a set of 29 discrete values of $A_i \in S$ ={0.157, 0.404, 0.384, 0.364, 0.331, 0.309, 0.324, 0.291,0.278, 0.269, 0.254, 0.232, 0.212, 0.207, 0.198, 0.122, 0.116, 0.104, 0.092, 0.087, 0.081,0.075, 0.063, 0.057, 0.054, 0.049, 0.046, 0.040, 0.026} (m^2). The allowable stress (compression and tension) and the maximum deflection at mid-span constraints are considered to be 150 MPa and 3.5 cm, respectively.

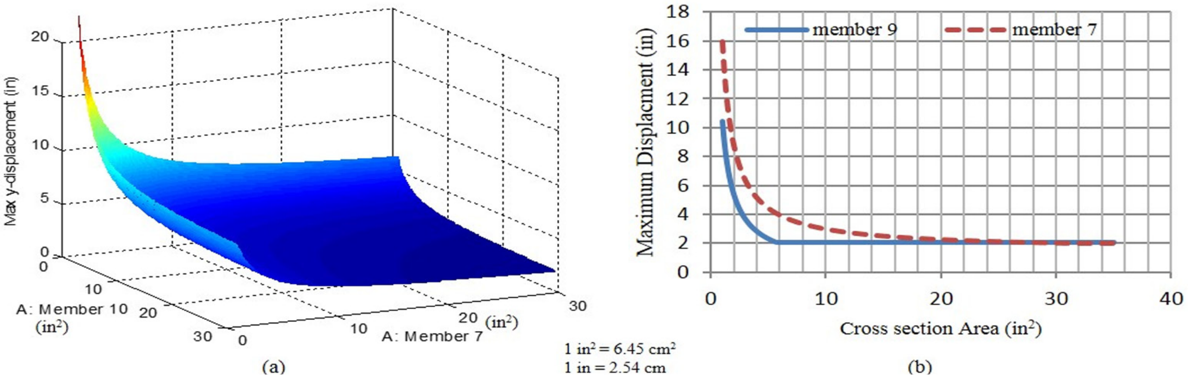


Fig. 3. Stress behaviour and displacement behaviour due to increasing the cross-sectional area of members 7 and 10

The bridge is assumed to be a dual-carriageway road with 2 lanes in each direction. The permanent load, including steel weight, surface and parapet, was 56 kN/m. The live load, including UDL and tandem system, was 45 kN/m. The maximum combination of the permanent load and live load was 127 kN/m by assuming the permanent and live factors as 1.20 and 1.35, respectively. Figure 4a shows typical deformations of the traditional analysis using the same cross-sectional area (0.257 m^2). Results obtained by the proposed discrete nonlinear approach are given in Table 4. The obtained results confirm that the proposed method not only reduces the structure weight but also significantly improves displacement and members stresses. It should be noted that the proposed algorithm leads to lighter weight, i.e., 21% less than traditional design. Also, the maximum displacement and maximum member stress ratio for discrete optimization was 16% and 33% less than the traditional design, respectively.

2.2.1. Convergence proof and sensitivity analysis

This section analyses the robustness and accuracy of the proposed method for the 61-bar Pratt truss bridge presented in Figure 4a. Optimum values of total weight and maximum deflection using initial population size as cross sectional area (1st to 5th run) are presented in Figure 4b. Numerical results indicate that the proposed method has better accuracy and robustness compared to the traditional design methods.

For better comparisons, Figure 5a shows the convergence curves of the Pratt truss as an average of four different runs and two cases. In the first case, sensitivity analysis was carried out for all members in the structure by increasing their cross-sectional areas by the ratio of $\partial A(x)/\partial x$, which was defined as 1 mm^2. In the second case, top and bottom chords are considered to have greater areas than braces areas, i.e., A+13 mm^2, A+3 mm^2 and A, respectively (see Fig. 4a). Sensitivity analysis was then carried out by increasing the cross-sectional area of the members A by the ratio of $\partial \mathrm{A}(\mathrm{x})/\partial \mathrm{x}$ equal to 1 mm^2. Figure 5b shows the maximum joint displacement with respect to weight, where the red and green lines correspond to the first and the second case, respectively. It is shown that the second case leads to better design solution in terms of displacement and weight compared to the first case.

2.3. Optimum design of 57-bar planar tower truss

The efficiency of the proposed method is studied for a large scale structures, the 57-bar space transmission

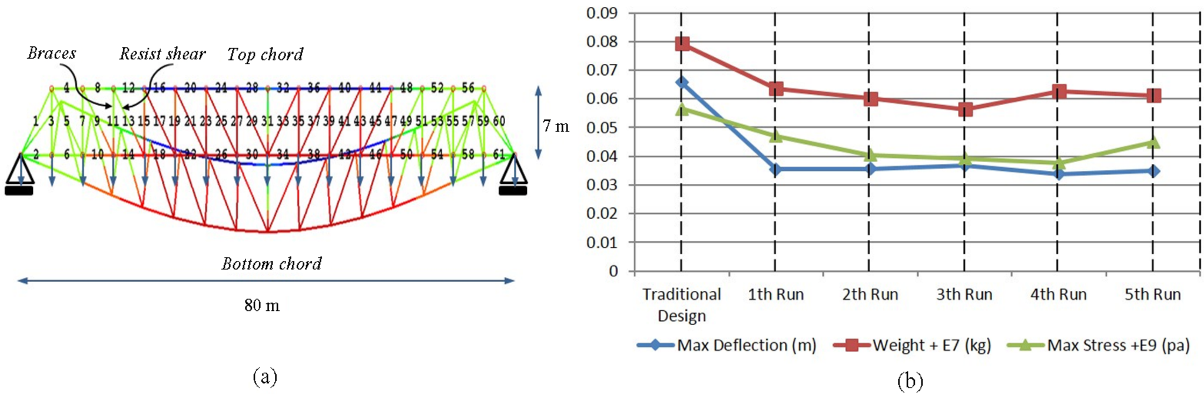


Fig. 4. (a) Geometry of 61-bar Pratt truss bridge and structural deformation of traditional design; (b) Comparison between optimum design solutions using five different sets of initial cross sectional areas (1st to 5th run) with the traditional design

Table 4. Optimum design results for the Pratt Truss Bridge using different types of design variables (Max *y*-displacement: 3.5 cm)

Design Variables (cm²)	Traditional Variables	Discrete Variables	Design Variables (cm²)	Traditional Variables	Discrete Variables
1	2570	3088	**32**	2570	3840
2	2570	3088	**33**	2570	486
3	2570	865	**34**	2570	2320
4	2570	2540	**35**	2570	571
5	2570	2320	**36**	2570	4040
6	2570	2776	**37**	2570	924
7	2570	2074	**38**	2570	2120
8	2570	3308	**39**	2570	625
9	2570	2908	**40**	2570	3840
10	2570	2074	**41**	2570	1158
11	2570	1158	**42**	2570	1217
12	2570	4040	**43**	2570	805
13	2570	3240	**44**	2570	3640
14	2570	745	**45**	2570	2120
15	2570	1570	**46**	2570	541
16	2570	4040	**47**	2570	1042
17	2570	1981	**48**	2570	3088
18	2570	865	**49**	2570	2320
19	2570	1981	**50**	2570	625
20	2570	3840	**51**	2570	1981
21	2570	924	**52**	2570	3240
22	2570	1570	**53**	2570	2688
23	2570	456	**54**	2570	2320
24	2570	4040	**55**	2570	2120
25	2570	865	**56**	2570	3088
26	2570	2074	**57**	2570	2688
27	2570	486	**58**	2570	3308
28	2570	4040	**59**	2570	625
29	2570	625	**60**	2570	2776
30	2570	2320	**61**	2570	3840
31	2570	401			
weight (Kg)				793135.0	626506.4
Displacement (cm)				4.01	3.38
Sigma (MPa)				56.476	37.798

tower shown in Figure 6a. All members were assumed to be made from the mild steel with E equal to 210 GPa, ρ equal to 7860 Kg/m^3, σ_{all} (compression and tension) equal to 147.15 MPa and $\delta_{\max}$ equal to 2.05 mm. Available cross-sectional areas corresponding with discrete optimization under the applied loads were taken from CE Marked Structural Sections (2013) using a set of 57 discrete values of $A_i \in S$ = {0.048, 0.057, 0.059, 0.063, 0.066, 0.069, 0.082, 0.082, 0.087, 0.094, 0.101, 0.11, 0.115, 0.119, 0.123, 0.132, 0.141, 0.143, 0.151, 0.155, 0.167, 0.179, 0.187, 0.192, 0.206, 0.212, 0.218, 0.227, 0.251, 0.254, 0.262, 0.264}×10^{-2} m^2.

For continues variable optimization, cross-sectional area of each member was selected in the range of 0.00001 to 0.1 m^2. Buckling analysis was performed for this example under external loads shown in Figure 6a to calculate the critical loads which can cause instability and collapse in the whole structure. The buckling load (i.e. critical load at which the structure would buckle) was calculated based on eigenvalue analysis, and determined as 2.04 kN. Figure 6b shows the structural deformation under design loads by taking into account the buckling effects. Table 5 compares the results of optimization using continues and discrete variables with the traditional design solution using uniform cross-sectional areas. It

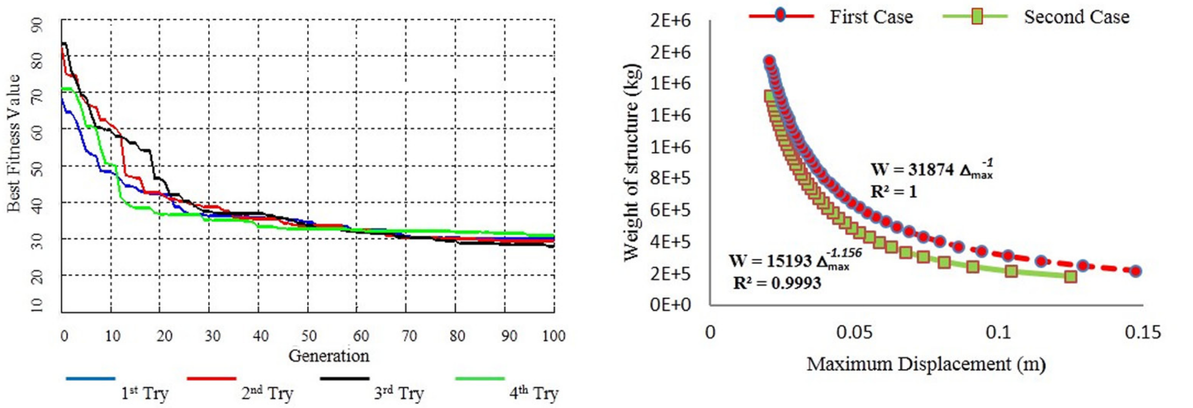
Table 5. Optimum design results for the 57-bar planner truss using continues and discrete variables

Design Variables (cm²)	Traditional Variables	Continuous Variables	Discrete Variables	Design Variables (m²)	Traditional Variables	Continuous Variables	Discrete Variables
1	11.5	8.31	15.5	**30**	11.5	8.75	14.1
2	11.5	7.77	4.8	**31**	11.5	7.66	5.86
3	11.5	7.66	5.69	**32**	11.5	8.68	6.91
4	11.5	8.41	11.9	**33**	11.5	8.61	9.4
5	11.5	8.82	4.8	**34**	11.5	8.49	11.9
6	11.5	8.18	21.2	**35**	11.5	8.06	5.86
7	11.5	8.63	8.24	**36**	11.5	7.83	6.31
8	11.5	8.88	6.31	**37**	11.5	8.69	25.4
9	11.5	8.67	20.6	**38**	11.5	8.91	6.56
10	11.5	8.84	14.3	**39**	11.5	8.26	8.7
11	11.5	8.68	26.4	**40**	11.5	5.14	13.2
12	11.5	7.64	10.1	**41**	11.5	8.87	6.56
13	11.5	8.84	11	**42**	11.5	7.66	5.86
14	11.5	8.37	21.8	**43**	11.5	8.9	8.7
15	11.5	7.9	12.3	**44**	11.5	8.07	26.4
16	11.5	8.81	18.7	**45**	11.5	8.16	12.3
17	11.5	8.54	15.1	**46**	11.5	7.97	8.7
18	11.5	8.67	11.9	**47**	11.5	8.03	19.2
19	11.5	8.59	21.2	**48**	11.5	8.67	8.24
20	11.5	7.93	14.1	**49**	11.5	8.91	6.56
21	11.5	7.96	8.7	**50**	11.5	8.91	9.4
22	11.5	7.73	5.69	**51**	11.5	7.91	13.2
23	11.5	8.35	21.2	**52**	11.5	7.69	9.4
24	11.5	7.56	8.7	**53**	11.5	5.97	11
25	11.5	7.93	13.2	**54**	11.5	8	18.7
26	11.5	8.94	16.7	**55**	11.5	8.01	6.56
27	11.5	8.55	6.91	**56**	11.5	8.26	6.31
28	11.5	8.06	5.69	**57**	11.5	8.62	14.1
29	11.5	7.97	13.2				
weight (Kg)					981.891	70.141	893.692
Displacement (cm)					1.47	1.99	1.07
Sigma (MPa)					0.8538	1.1940	0.4919

is shown that optimization of truss topology using continuous and discrete variables leads to 29% and 9%, respectively, savings in the total structural weight when compared to the conventional design. Furthermore, Table 5 shows that the optimal design based on discrete optimization approach resulted in 27% and 42% less displacement and member stress compared with the traditional design, which highlights the efficiency of the proposed method in practical applications. Figure 6c presents the maximum nodal displacement of the 57-bar planar tower truss as a function of structural weigh. Through fitting curve approach, it can be observed that $W \times \Delta_{max} = 14.46$ and correspondingly, $W \times \sigma_{max} = 8 \times 10^8$. As a result, $\frac{\sigma_{max}}{\Delta_{max}}$ has a constant value of 5.532×10^7. This means that there is a linear relationship between the maximum nodal displacement and maximum member's stress, which can be used for practical design purposes.

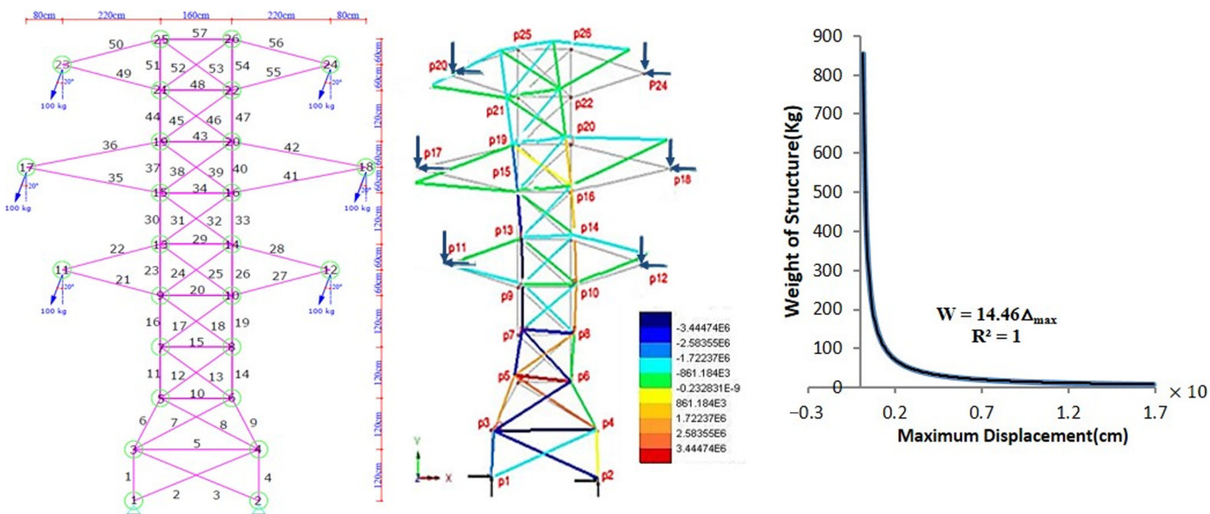
Summary and conclusions

In this study, modified Augmented Lagrangian Genetic Algorithm (ALGA) and Quadratic Penalty Function Genetic Algorithm (QPGA) optimization methods were developed for size and topology optimization of truss structures to obtain acceptable design solutions with minimum structural weight. The proposed method was validated by optimizing a 10-bar truss structure, a Pratt



(a) (b)

Fig. 5. Typical convergence curves of the optimization model (a) Maximum joint displacement sensitivity of Pratt truss bridge (b)



(a) (b) (c)

Fig. 6. (a) The geometry of the 57-bar planner truss; (b) Structural deformation with considering buckling analysis; (c) Maximum nodal displacement sensitivity analysis

truss bridge and a 57-bar planar tower truss using both discrete and continuous variables. The numerical examples verified the feasibility of the developed algorithm, and indicated that the adopted method can significantly reduce the structural weight and maximum deflection of the conventional design. It was shown that the optimal design solution for the Pratt truss bridge using discrete optimization is 21% lighter than the traditional design, while it will also exhibit 16% and 33% less maximum joint displacement and maximum member stress under the design loads, respectively. The size and topology optimization of the 57-bar planar tower truss through continuous and discrete optimization also resulted in 29% and 9% lighter structures than traditional design, respectively. Several sensitivity analyses were conducted to show the robustness and reliability of the proposed optimization methods, which should prove useful in optimum design of large-scale truss structures.

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