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AN INVESTIGATION OF ABF++, LSCM, AND ARAP METHODS FOR PARAMETRIZATION OF SHOETREES

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Abstract. This paper discusses on the numerical model flattening algorithms enabling the representation of straightened surface of three-dimensional objects in the plane. These algorithms are widely used in the engineering industry which requires a precise representation of surfaces of various fragments of 2D maps, such as the automotive industry. One of the examples of the application in the manufacture of footwear could be automatic formation of molds using shoetrees, which are usually produced by a unit ignoring the fact that manufacturers are still using wax foil for flattening. This paper provides an investigation of the most widely used 3D object surface flattening algorithms and also the comparison of molds of shoetrees obtained by these methods.

Keywords: 3D objects, 3D scanning, flattening algorithms, shoetrees.

Introduction

Recent innovation in 3D acquisition technology, such as computer tomography or 3D laser scanning, enabled highly accurate digitalization of complex 3D objects. So efficient algorithms able to preprocess and handle these objects are needed. Invention of 3D scanning technology played an important role in new fields of application research and development regarding 3D data analysis (Telfer, Woodburn 2010; Rodriguez-Quinonez *et al*. 2011). Fast-growing mass customization markets require new fields of research to improve manufacturing efficiency regarding unique products (Pataky *et al*. 2011). Digital drawings and 3D models provide 70% of technical data (Luximon, A., Luximon, Y. 2012; Patrikalakis, Maekawa 2000) that is needed for clear manufacturing, however there are still some unused applications where 3D data can be applied to operate with complete efficiency. Innovative solutions are needed to develop an automatic system that will flatten the surface of the foot's complex geometry. Successful results can be applied to different fields of nowadays manufacturing, such as: clothes industry, furniture industry, medical equipment industry, automotive industry and especially for virtual design based on operating with individual scanned data to apply efficient manufacturing of custom products. Developed a complex digital 3D geometry surface and outer shell modeling to a planar surface method that can be used in more than just footwear manufacturing. This automated manufacturing process could be widely adapted for individual orthopedic and other medicinal equipment, automotive and wheelchair seats, clothing, furniture and other possible branches of manufacturing that requires a perfect fit to a person's specific form.

Today all acting solutions are focused on serial production (Azariadis, Papagiannis 2010). Digital modeling processes in serial production are very clear and accurate however technology requires 70–80 % set-up time (Kolisch 2000). It is quite difficult to embed virtual design and data processing systems, adapted for serial production, to mass customization manufacturing processes. Considering to custom output manufacturing setup time requires too much time comparing with serial production. Accordingly in custom production main processes still are performed by humans. Currently in custom footwear production flattening of individual lasts is a manual job. There are a number of decisions in market relating to the automated manufacturing of footwear. Decisions are implemented in separated systems, or realized as a universal plug-ins.

Flattening algorithms

There are many 3D modelling programs, which have flattening functions, for example ShoeMaster, Blender 3D, Rhinoceros, 3D coat. Unfortunately, flattened shells of non-standard shoetrees by these programs do not match a technical data of flattened shells, obtained by digitization of wax foil. To compute a flattened surface of a given 3D object (or graph) means to construct an isomorphic graph on plane. One of the main applications of that construction is texture mapping. The parameterization is used to put the surface into one-to-one correspondence with an image, stored in the 2D domain. There are many methods for surface flattening, called mesh parametrization, such as pure methods: authalic (area-preserving) mapping (Alliez *et al*. 2002), conformal (angle-preserving) mapping (Levy *et al*. 2002), isometric (length-preserving) mapping (Liu *et al*. 2008) and mixed methods which are combinations of these.

The main goal for footwear modelling is to minimize stretching of leather. That's means area preserving mapping is inappropriate, because it deforms a surface mostly. So for experimentation chosen methods are: pure conformal mapping, pure isometric mapping and angle based flattening, that is closely related to conformal mapping.

Angle based flattening

Angle based flattening (ABF) is a method of mapping that preserves similarity of triangles of a given mesh to corresponding triangles of flattened mesh (Sheffer, Sturler 2001). ABF minimizes the augmented objective function F:

$$F\left(\alpha,\lambda_1,\lambda_2,\lambda_3\right)=$$
$$\sum_{t\in T}\sum_{k=1}^{3}\left(\frac{1}{w_k^t}\left(\alpha_k^t-\beta_k^t\right)+\lambda_1^t C_1\left(t\right)\right)+\sum_{v\in V}\sum_{i=2}^{3}\lambda_i^v C_i\left(v\right), \quad (1)$$

where T is a set of indexed triangles, V is a set of indexed vertices α_k^t are unknown edges λ_1, λ_2, λ_3 are unknown Lagrange multipliers, α_k^t are given edges and $\alpha_k^t=\left(\beta_k^t\right)^{-2}$ are weights, which reflects relative rather than absolute angular distortion. There are three more constraints for planar parametrization:

1. Triangle validity: $C_1\left(t\right)=\alpha_1^t+\alpha_2^t+\alpha_3^t-\pi$,
2. Planarity: $C_2\left(v\right)=\sum_{(t,k)\in v^*}\alpha_k^t-2\pi$, where v^* is the set of angles incident on vertex v,
3. Reconstruction:

$$C_3\left(v\right)=\prod_{(t,k)\in v^*}\sin\alpha_{k\oplus 1}^t-\prod_{(t,k)\in v^*}\sin\alpha_{k\ominus 1}^t.$$

To minimize the function $F\left(x\right)=F\left(\alpha,\lambda_1,\lambda_2,\lambda_3\right)$, ABF algorithm uses Newton's method:

while $\|\nabla F\left(x\right)\|>\epsilon$ do
  solve $\nabla^2 F\left(x\right)\delta=-\nabla F\left(x\right)$
  $x\leftarrow x+\delta$
end while

There is the same formulated problem for algorithm ABF++ (Sheffer *et al*. 2005). The difference between ABF and ABF++ algorithms is that ABF++ method based on mathematical results that dramatically improve the performance for solving linear system $\nabla^2 F\left(x\right)\delta=-\nabla F\left(x\right)$.

Least-squares conformal mapping

The least-squares conformal mapping (LSCM) parameterization is an angle preserving algorithm that generates a discrete approximation of a conformal map (Levy *et al*. 2002). LSCM minimizes the conformal energy $\varepsilon_C\left(S\right)$ of the mapping X, defined by:

$$\varepsilon_C\left(S\right)=\int_S\left\|\nabla v-(\nabla u)^{\perp}\right\|=\int_S\left|\frac{\partial X\left(t\right)}{\partial v}-i\frac{\partial X\left(t\right)}{\partial u}\right|^2 dt, \quad (2)$$

where S is an area of the surface, $\nabla v=(\nabla u)^{\perp}$ are Cauchy-Riemann equations and $X:\mathbb{R}^3\to\mathbb{C}$, $\left(x,y,z\right)\mapsto u+iv$ is the inverse of parametrization. For a piecewise linear parametrization, the conformal energy ε_C expressed as a function in a parameter-space coordinates $\left(u_k,v_k\right)$. These 2D coordinates could be calculated using the Conjugate Gradient method.

As rigid as possible

Another group of flattening methods belongs to methods that are preserving edges of surface triangles. Most widely used algorithm for length-preserving is as rigid as possible (ARAP) that generates a projection of surface in the plane by keeping triangles as rigid as possible (Liu *et al*. 2008). Consider an energy function:

$$E\left(u,L\right)=\sum_{t=1}^{T}A_t\left\|J_t\left(u\right)-L_t\right\|_F^2=$$
$$\sum_{t=1}^{T}A_t\ \mathrm{tr}\left(\left(J_t\left(u\right)-L_t\right)^T\left(J_t\left(u\right)-L_t\right)\right), \quad (3)$$

where $x_t=\left\{x_t^0,x_t^1,x_t^2\right\}$ are triangles of 3D mesh, $u_t=\left\{u_t^0,u_t^1,u_t^2\right\}$ are triangles of flattened mesh, A_t – area of 3D triangle $t\in T$, $J_t\left(u\right)$ – 2×2 Jacobian matrix which defines linear distance between triangles x_t and u_t, $\mathrm{L}\left(t\right)$ – 2×2 transformation matrix between triangles x_t and u_t, $\|\bullet\|_F$ is the Frobenius norm. ARAP algorithm minimizes $E\left(u,L\right)$ by minimizing following function:

$$\sum_{t=1}^{T}A_t\left(\left(\sigma_{1,t}-1\right)^2+\left(\sigma_{2,t}-1\right)^2\right), \quad (4)$$

where $\sigma_{1,t}$ and $\sigma_{2,t}$ are signed values of the Jacobian J_t of the *t*-th triangle's transformation.

Experimental investigation

To test the overlooked algorithms we have chosen five different digitized pairs of shoetrees with correspondent to molds which made by wax foil. After appreciation of correspondent molds of shoetrees, each shoetree was processed by cutting down a sole and head and then cutted straight from the middle of a heel to the middle of the second foot's finger (see Fig. 1).

In total 20 fragments were flattened by ABF++, LSCM and ARAP algorithms. An error of each fragment was calculated by $2C/(A+B)$, where A is area of original mould, B is area of flattened half-shoetree, C is the maximum area which can be obtained by intersection of original mould and flattened half-shoetree. A, B and C values were calculated by counting colored pixels of intersection of molds (Table 1–3). We've measured also differences between corner points, called margins (see Fig. 2). The dotted line corresponds to original mould, the solid line corresponds to boundary edges of flattened surface of shoetree. The values in *Margins* column in Table 1–3 coincides to difference in millimeters between corner points, numbered from 1 to 3 in Fig. 2.

Fig. 1. View of divided shoetree

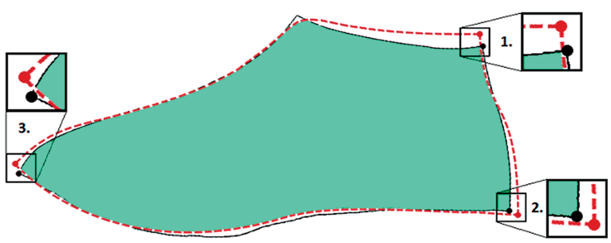

Fig. 2. Comparison of molds of shoetrees

Table 1. Flattening results using ABF++ algorithm

Pair No.	Shoetree	Side	A	B	C	2*C*/(*A*+*B*)	Margins, mm
1	L	L	578734	558968	558795	98.23 %	6+4+7
		R	570831	549012	524288	93.63 %	16+10+18
	R	L	584171	564993	565251	98.37 %	12+5+7
		R	574593	556319	547799	96.87 %	3+19+4
2	L	L	632804	619086	613302	97.98 %	4+12+3
		R	637810	609969	603171	96.67 %	10+23+21
	R	L	638950	611153	600192	96.02 %	13+9+10
		R	632804	613351	607965	97.57 %	16+16+12
3	L	L	550236	534187	527929	97.36 %	6+16+3
		R	539557	522001	521194	98.19 %	5+4+5
	R	L	547489	533744	532366	98.47 %	2+3+6
		R	539960	519292	512104	96.69 %	21+6+7
4	L	L	591323	573492	566721	97.30 %	7+5+5
		R	584236	562025	559188	97.56 %	7+5+15
	R	L	588692	573845	571064	98.24 %	9+6+3
		R	582918	564142	555682	96.88 %	8+4+21
5	L	L	505229	514098	505060	99.09 %	5+7+2
		R	513236	502834	500893	98.59 %	5+9+4
	R	L	511584	508920	496786	97.36 %	9+7+3
		R	508042	507705	501837	98.81 %	8+4+3

Table 2. Flattening results using LSCM algorithm

Pair No.	Shoetree	Side	A	B	C	2*C*/(*A*+*B*)	Margins, mm
1	L	L	578734	558968	558795	98.23 %	6+4+7
		R	570831	549012	524288	93.63 %	16+10+18
	R	L	584171	564993	565251	98.37 %	12+5+7
		R	574593	556319	547799	96.87 %	3+19+4
2	L	L	632804	619086	613302	97.98 %	4+12+3
		R	637810	609969	603171	96.67 %	10+23+21
	R	L	638950	611153	600192	96.02 %	13+9+10
		R	632804	613351	607965	97.57 %	16+16+12
3	L	L	550236	534187	527929	97.36 %	6+16+3
		R	539557	522001	521194	98.19 %	5+4+5
	R	L	547489	533744	532366	98.47 %	2+3+6
		R	539960	519292	512104	96.69 %	21+6+7
4	L	L	591323	573492	566721	97.30 %	7+5+5
		R	584236	562025	559188	97.56 %	7+5+15
	R	L	588692	573845	571064	98.24 %	9+6+3
		R	582918	564142	555682	96.88 %	8+4+21
5	L	L	505229	514098	505060	99.09 %	5+7+2
		R	513236	502834	500893	98.59 %	5+9+4
	R	L	511584	508920	496786	97.36 %	9+7+3
		R	508042	507705	501837	98.81 %	8+4+3

Table 3. Flattening results using ARAP algorithm

Pair No.	Shoetree	Side	A	B	C	$2C/(A+B)$	Margins, mm
1	L	L	578734	564603	551139	96.40 %	17+16+9
		R	570831	563310	556483	98.13 %	13+14+4
	R	L	584171	570637	559070	96.82 %	9+6+7
		R	574593	569054	557354	97.46 %	9+16+9
2	L	L	632804	611394	614813	98.82 %	4+17+7
		R	637810	612580	612370	97.94 %	14+10+13
	R	L	638950	611928	603507	96.49 %	15+3+11
		R	632804	613421	612873	98.35 %	12+5+2
3	L	L	550236	530668	527524	97.60 %	8+4+2
		R	539557	524405	521426	98.01 %	12+13+28
	R	L	547489	534803	534732	98.81 %	7+7+4
		R	539960	517571	499708	94.50 %	17+14+21
4	L	L	591323	574939	566718	97.18 %	5+2+7
		R	584236	566640	566392	98.42 %	18+11+9
	R	L	588692	576627	568218	97.52 %	25+6+25
		R	582918	561662	552297	96.50 %	9+2+9
5	L	L	505229	505701	498905	98.70 %	8+4+6
		R	513236	511408	505028	98.57 %	11+13+15
	R	L	511584	508521	504457	98.90 %	7+4+7
		R	508042	506398	501156	98.80 %	5+2+9

Conclusions

In this paper an investigation of flattening algorithms was presented. Experimental results has showed that average values of relative similarity of flattened half-shoetrees equal to 97.49%, 97.36%, 97.70%, which were obtained by ABF++, LSCM and ARAP algorithms, respectively.

According it is advisable to use ARAP algorithm for shoetree flattening. However, algorithms were compared interdependently and according results there still wasn't suitable evolvents for shoe production.

The main thing is that difference between corner points fluctuates middling about 10 mm, though permissible error is only 2 mm. Therefore, future works on this problem can be testing of mixed parametrization algorithms or flattening by 3D contour.

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KURPALIŲ IŠKLOTINIŲ SUDARYMO TAIKANT PARAMETRIZAVIMO METODUS ABF++, LSCM, ARAP TYRIMAS

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Santrauka

Apžvelgiami skaitmeninių modelių paviršių išklotinių sudarymo algoritmai ABF++, LSCM, ARAP, pagal kuriuos trimačių objektų paviršiai atvaizduojami ištiesinti plokštumoje. Šie išklotinių sudarymo algoritmai plačiai taikomi ne tik 3D skaitmeninių modelių tekstūroms generuoti bei atkurti, bet ir pramonės inžinerijoje, kur būtina įvairių detalių paviršių fragmentus tiksliai atvaizduoti plokštumoje. Vienas iš taikymo pavyzdžių avalynės gamyboje galėtų būti automatinis lekalų sudarymas pagal įvairius kurpalių modelius, kurie dažniausiai gaminami vienetiniams gaminiams. Iki šiol paviršiaus išklotinei sudaryti naudojama vaško folija. Atliktas tyrimas parodė, kad iš kurpalių išklotinių, gautų remiantis teoriniais metodais, geriausių rezultatų pasiekta taikant ARAP algoritmą, tačiau jis nėra pakankamai tikslus, kad būtų galima tiesiogiai taikyti avalynės gamyboje.

Reikšminiai žodžiai: 3D objektai, 3D skenavimas, paviršių išklotinių sudarymo algoritmai, kurpaliai.