



REMANUFACTURING WITH PATENTED TECHNIQUE ROYALTY UNDER ASYMMETRIC INFORMATION AND UNCERTAIN MARKETS

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Abstract. We study a dual-channel recycling closed-loop supply chain (CLSC) and investigate the royalty strategy involving cost-reducing technique for remanufacturing patented products. Facing information asymmetry and market uncertainty, we address the problem where the patent licensor (manufacturer) and licensee (remanufacturer) simultaneously compete in the sales market and the recycling market. We examine the optimal decisions of a decentralized CLSC (D-CLSC) with the manufacturer being the Stackelberg leader. Numerical examples are used to demonstrate how the patented technology (cost-reducing technique) affects the channel players' behaviors and how to identify the optimal royalty fee. Based on the theoretical derivation and the numerical outcomes, we find that regardless of the CLSC structure (centralized or decentralized), the take-back prices and the total profits will rise with the increase of savings from the licensed technology. In the D-CLSC, (i) the expected profits of the manufacturer and the remanufacturer as well as the royalty fee will also rise with the savings from the licensed technology. (ii) In addition, the wholesale price, retail price, take-back prices, as well as the royalty fee will rise with the degree of information asymmetry. But the retailer's expected profit will decline. (iii) Finally, the expected profit of the manufacturer will rise with the demand uncertainty and the return uncertainty. For the remanufacturer, this trend is not obvious. Our research provides guidance to resolve conflicts and intellectual property disputes between the original manufacturer and the remanufacturer of the patented product.

Keywords: closed-loop supply chain, remanufacturing, fuzzy decision, royalty licensing, game theory.

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Introduction

The environment pollution, resource limit, and population growth are three major challenges facing the current society. The environment and resources are particularly critical for sustainable development of economy and society, and directly affect the quality of human life (Khaksar, Abbasnejad, Esmaili, & Tamošaitienė, 2016; Bai & Sarkis, 2018). Therefore, protecting the ecological environment and rationally utilizing resources are essential for creating a sustainable environment. Many countries have strived to promote the green manufacturing through recycling products. For example, the US, Germany, Netherlands and Japan have successively implemented the Extended Producer Responsibility (EPR) to deal with the rapid growth of car ownership and the drastic increase in scrapped cars. Low carbon economy has thus increasingly attracted worldwide attention, as a way to cope with global warming and energy depletion. Companies worldwide are trying to combine low-carbon and economic benefits to develop a new growth direction. Closed-loop supply chain (CLSC) offers a new opportunity for firms to win green reputation, reduce costs and increase profits. For instance, Xerox Corp. implemented a green remanufacturing program that saves 40–65% on manufacturing costs through the reuse of parts and materials (Savaskan, Bhattacharya, & Van Wassenhove, 2004). Hewlett-Packard Corp. adopted a similar approach for their computers and peripherals, while Canon also carried out similar activities for printing and copying ink cartridges.

With the maturity of product recycling channels and the development of remanufacturing technologies, specialized remanufacturers have emerged in the market, such as Lexmark, Cardone, etc. In some industries, the distributors of the manufacturer are also engaged in remanufacturing. For example, to enhance its capacity, Caterpillar Corp. contracts with an independent remanufacturer to help collect the used products and remanufacture. However, it requires the original manufacturer's patented technical support and brand royalty. With the growth of the remanufacturing industry, the conflicts of interest and patent disputes between the original manufacturers and the remanufacturers are increasing. For example, Canon has filed numerous patent lawsuits on remanufactured ink cartridges (Hashiguchi, 2008). Other patent related remanufacturing litigations include Cotton Tie Co. vs. Simmons; Sandvik Aktiebolag vs. E. J. Co., etc. (Liu, 2014).

In general, patented licensing has two opposite effects on patent holders. On the one hand, it can increase its profit margin by collecting the royalty fee (abbrev. royalty) of patented technology, which can be seen as the income effect. On the other hand, the licensing would increase market competition and erode the profit margins of manufacturers, and thus, it can be seen as the competition effect. This is also a problem which should be considered in remanufacturing research. Arora and Ceccagnoli (2006) analyzed the relationships between the patent protection, complementary assets, and firms' incentives for technology licensing. And then, many researchers (Lin & Kulatilaka, 2006; Brousseau, Coeurderoy, & Chaserant, 2007; Nagaoka, 2009; Zhao, Chen, Hong, & Liu, 2014, etc.) have discussed deeply on product's optimal licensing strategy, and focused on the manufacturing of new products. There is little work on patented licensing issues regarding remanufacturing. Oraipoulos, Ferguson, and Toktay (2012) studied the optimal relicensing strategy for original equipment

manufacturers (OEMs) in both monopoly and the duopoly market. In their study, a third-party purchased the used products from the OEM's customers, refurbished them, and resold them in competition with the OEM's new products. Recently, Zhang, and Ren (2016) investigated coordination strategy for remanufacturing patented products with different retail prices. Hong, Govindan, Xu, and Du (2017) discussed two technology licensing strategies and the optimal production and collection decisions in a CLSC.

The above literature is based on the symmetry of information and does not consider the uncertainty of the market demand and return. The parameters such as demand, manufacturing/remanufacturing costs, and recycling costs are all assumed to be certain. In fact, the supply chain members often protect their own interests and ensure their own needs, costs, and other information remain confidential. Thus, it is not easy for other companies to attain the information. For example, due to the complexity of recycling, manufactures cannot clearly know the exact recycling costs of the remanufacturer. In addition, the remanufacturing cost will also be difficult to obtain because of the different qualities of the used products. Few researchers have explored CLSC under asymmetry information. Zhang, Y. Xiong, Z. Xiong, and Yan (2014) designed contracts for CLSC when the collection cost is the retailer's private information. Wei, Govindan, Li, and Zhao (2015) studied pricing and collecting decisions in a CLSC when the manufacturing and remanufacturing costs are the manufacturer's private information, while the market base and collecting scale are that of the retailer.

Besides, uncertainties in demand and used-product return are two major sources of CLSC risks. Shi, Zhang, Sha, and Amin (2010), Shi, Zhang, and Sha (2011) investigated stochastic demand and returns to coordinate production and recycling decisions. Amin and Zhang (2013) developed a CLSC facility location model under uncertain demand and return. Then again, Khatami, Mahootchi, and Farahani (2015), Mohammed, Selim, Hassan, and Syed (2017) and Cui et al. (2017) designed a CLSC network for uncertain demand and return. Giri and Sharma (2016), Kim, Do Chung, Kang, and Jeong (2018) researched the production strategy for CLSC with uncertain demand and return. Also, Hamdouch, Qiang, and Ghoudi (2017) proposed a CLSC equilibrium model with random and price-sensitive demand and return.

The above study used the known probability distribution of parameters to characterize the uncertainty in the CLSC. However, with the rapid development and intensification of global economy, the manufacturing environments are often subject to dramatic changes and frequent disruptions. Moreover, continual fluctuation in international exchange rates and increasingly fierce market competition make it difficult to obtain reliable market data to accurately estimate parameters under study. Using a specific probability distribution (e.g. Normal distribution, Poisson distribution) to describe the unknown parameters may result in significant errors. Zimmermann (2000) pointed out that when the decision makers (DMs) lack historical data, parameter estimates based on expert knowledge or experience are more appropriate. For example, the linguistic terms "high", "medium" or "low" can be used to estimate the remanufacturing cost or the recycling cost. For the market demand, the linguistic terms "large" or "small" could be used.

Fuzzy theory (Zadeh, 1996) provides a reasonable and effective method to deal with this kind of uncertain optimization problem (Liu & Xu, 2014; Xu, Patnayakuni, Tao, & Wang,

2015; Keshavarz Ghorabae, Amiri, Olfat, & Khatami Firouzabadi, 2017; Yildizbaşı, Çalik, Paksoy, Farahani, & Weber, 2018). For example, when describing the cost, the DMs can give their judgements as “around x dollars”, which is a fuzzy variable \tilde{x} . Many researchers in recent times have used fuzzy theory to depict the uncertainty in the CLSC. Wei and Zhao (2011, 2013) explored pricing decisions with retail competition in a fuzzy CLSC. They also investigated three different reverse channel decisions for fuzzy CLSC, involving fuzziness in demand, remanufacturing and collection costs. Zarandi, Sisakht, and Davari (2011) presented an interactive fuzzy goal programming model for CLSC optimization by considering uncertainty in the DMs’ aspiration levels. Ramezani, Kimiagari, Karimi, and Hejazi (2014) proposed a CLSC design model, taking into account the fuzziness of constraints, the lack of knowledge and DM’s goal. Finally, Zhao, Wei, and Sun (2016) investigated two coordinating models with symmetric and asymmetric information of fuzzy CLSC. However, the above research on fuzzy CLSC only focuses on (re)manufacturer’s production decisions. They did not consider the impact of the patented royalty strategy and the simultaneous competition of (re)manufacturer in the sales and recycling markets. The coordination problem of CLSC with patented royalty with asymmetric information and dual uncertain markets (uncertain demand and uncertain return) has not been studied. The main differences between this study and the previous related literature are contrasted in Table 1.

In this research, we focus on a dual-channel recycling CLSC problem, where the manufacturer and the remanufacturer directly collect the used product from customers. In the model, the manufacturer is a patent licensor who produces new products as well as remanufactured products. Remanufacturer, as a licensee, can only produce the remanufactured products. In fact, when the patent licensor also participates in the recycling and remanu-

Table 1. A summary of differences between this study and the previous literature

Literature	Patent License for Remanufacturing of Patented Products	Asymmetric Information	Dual Uncertain Markets	Fuzzy Theory
Oraiopoulos et al., 2012; Zhang and Ren, 2016; Hong et al., 2017	√			
Zhang et al., 2014; Wei et al., 2015		√		
Shi et al., 2010; Shi et al., 2011; Amin and Zhang, 2013; Khatami et al., 2015; Mohammed et al., 2017; Hamdouch et al., 2017			√	
Wei and Zhao, 2011; Wei and Zhao, 2013; Zarandi et al., 2011; Ramezani et al., 2014				√
Zhao et al., 2016		√		√
This paper	√	√	√	√

facturing of the used products, the licensor and the licensee not only compete in the sales market, but also compete in the recycling market. The direct dual-channel competition is beneficial to consumers, contributes to the performance improvement of the CLSC recycling and better protection of the environment. The focus of this research can be summarized as follows:

- 1) We study the royalty strategy for the cost-reducing technique, where the licensor and licensee compete in the sales market and the recycling market.
- 2) We consider the information asymmetry in CLSC, that is, the supply chain members have incomplete information on each other’s production costs. We also study the influence of asymmetric information on the optimal decision and coordination of the CLSC.
- 3) We conduct research in the dual uncertain environment where the market demand for new products is uncertain and the market supply of the used products is uncertain. The objective is to maximize expected profits and coordinate the optimal decisions of each member in the CLSC.

The remainder of this paper is organized as follows: Section 1 introduces the model assumption and defines the notations necessary for this research. The model formulations and the solutions are presented in Section 2. Section 3 provides some numerical examples to further compare the results established in the two different decision scenarios, and to explore how cost-saving from the patented manufacturing technique (hereafter cost-saving) will affect the channel player’s behaviors and the royalty strategy of the CLSC. In Section 4, we analyze and discuss the behavior of CLSC members when facing information asymmetric and uncertain markets. Concluding remarks are given in last Section.

1. Model specifications and parameter definition

We consider a supply chain consisting of a manufacturer (the patent licensor), a remanufacturer (the patent licensee) and an independent retailer with information asymmetry and market uncertainty. The CLSC structure is shown in Figure 1. The manufacturer produces new products as well as remanufactured products, while the remanufacturer only produces remanufactured products. The manufacturer and remanufacturer have their own private information, such as the exact manufacturing cost and remanufacturing cost, and unwilling to release the sensitive information to each other. They all collect the used products directly through their own collection channels, and assume that all the collected used products can be processed for remanufacturing.

Considering the information asymmetric structure, the manufacturer produces the new patented product with unit manufacturing cost \tilde{c}_n or from the collected used product with unit

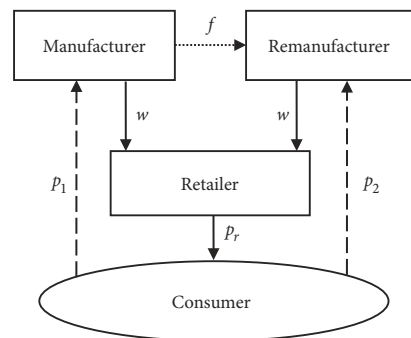


Figure 1. Structure model of the CLSC

remanufacturing cost \tilde{c}_r . \tilde{c}_{r_2} is the unit cost of the remanufacturer in remanufacturing a used product. The parameters \tilde{c}_n , \tilde{c}_{r_1} and \tilde{c}_{r_2} are nonnegative fuzzy variables. Suppose that the manufacturer is superior to the remanufacturer in technology, that is $E[\tilde{c}_{r_1}] < E[\tilde{c}_{r_2}]$. The manufacturer then licenses its cost-reducing technique to remanufacturer and charges a unit royalty fee f . After using the manufacturer's licensed technology, the remanufacturer reduces the remanufacturing cost to the manufacturer's cost level. Suppose $\tilde{c}_r = \tilde{c}_{r_2} = \tilde{c}_r$ and let $\tilde{c}_\Delta = \tilde{c}_n - \tilde{c}_r$, where \tilde{c}_Δ is the unit cost-saving for manufacturer through remanufacturing.

In addition to the above mentioned, the following parameters will be used throughout the paper:

- p_r : Unit retail price of the new product, which is the retailer's decision variable, $p_r \geq 0$.
- w : Unit wholesale price set by the manufacturer for the retailer, which is the manufacturer's decision variable. After using the manufacturer's cost-reducing technique, the remanufacturer reduces its unit production costs and sells its products to retailer at the same unit wholesale price as w , $w \geq 0$.
- p_1 : The manufacturer's unit take-back price of the used products, which is the manufacturer's decision variable, $p_1 \geq 0$.
- p_2 : The remanufacturer's unit take-back price of the used products, which is the remanufacturer's decision variable, $p_2 \geq 0$.
- D : The total market demand for new products, which is a linear function of the retail price p_r . Consistent with the existing literature (Savaskan et al., 2004), we assume that new products and remanufactured products are the same in quality and are equally accepted by consumers. The market demand function is $D(p_r) = \tilde{\phi} - \tilde{\beta}p_r$, where $\tilde{\phi}$ denotes the market capacity and $\tilde{\beta}$ denotes the price sensitivity coefficient. Here, the parameters $\tilde{\phi}$ and $\tilde{\beta}$ are non-negative fuzzy variables because the market demand is usually uncertain.
- S : The market supply of the used products. Referring to Park and Keh (2003), we assume that the market supply functions of the used products are $S_i(p_i, p_j) = \tilde{a} + p_i - \tilde{b}p_j$, $i \neq j$ and $i, j = 1, 2$, which denote the manufacturer and the remanufacturer respectively. \tilde{a} and \tilde{b} are non-negative fuzzy variables because the market supply of the used products is usually uncertain. \tilde{a} refers to the quantity of the used products that the consumers are willing to return when the take-back price is zero. It is related to the consumers' environmental awareness. The higher the environmental awareness of the consumers, the larger the value of \tilde{a} . \tilde{b} represents the substitution degree between the two collecting channels.
- π_j^i : The profits gained by the supply chain member j in the model i , in which $i = D, M$ denote the centralized CLSC (C-CLSC) and the decentralized CLSC (D-CLSC) respectively, $j = m_1, m_2, r, T$ denote the manufacturer, the remanufacturer, the retailer and the CLSC system respectively.

Throughout the paper, we make the following modeling assumptions:

Assumption 1. The parameters \tilde{c}_Δ , $\tilde{\phi}$, $\tilde{\beta}$, \tilde{a} , \tilde{b} are all independent nonnegative fuzzy variables. The fuzzy variable \tilde{c}_Δ satisfies $E[\tilde{c}_\Delta] > 0$, which means that using the collected used product to produce a new product is always more cost-effective than using the original

raw materials. The fuzzy variable \tilde{b} satisfies $0 < E[\tilde{b}] < 1$, which denotes that the quantity of the returned products collected is more sensitive to the collector’s own take-back price than the competitor’s take-back price.

Assumption 2. In the D-CLSC, we assume that the manufacturer is the leader in the Stackelberg game and the remanufacturer and the retailer are the followers.

From the above specifications and assumptions, the expected profits of the manufacturer, the remanufacturer, and the retailer can be written as follows:

$$\begin{aligned}
 E[\pi_{m1}] &= E\left[(w - \tilde{c}_n)[D(p_r) - S_2(p_1, p_2)] + (\tilde{c}_\Delta - p_1)S_1(p_1, p_2) + f S_2(p_1, p_2)\right] = \\
 &= E\left[(w - \tilde{c}_n)\left[\tilde{\phi} - \tilde{\beta}p_r - (\tilde{a} + p_2 - \tilde{b}p_1)\right] + (\tilde{c}_\Delta - p_1)(\tilde{a} + p_1 - \tilde{b}p_2) + f(\tilde{a} + p_2 - \tilde{b}p_1)\right] = \\
 &\left(E[\tilde{b}]w + E[\tilde{c}_\Delta] - E[\tilde{a}] - \frac{1}{2}\int_0^1(\tilde{c}_{n\alpha}^U \tilde{b}_\alpha^L + \tilde{c}_{n\alpha}^L \tilde{b}_\alpha^U)d\alpha\right)p_1 - \\
 &\left(w - E[\tilde{c}_n] + \frac{1}{2}\int_0^1(\tilde{c}_{\Delta\alpha}^U \tilde{b}_\alpha^L + \tilde{c}_{\Delta\alpha}^L \tilde{b}_\alpha^U)d\alpha\right)p_2 + \\
 &\left(E[\tilde{c}_n\tilde{\beta}] - E[\tilde{\beta}]w\right)p_r - p_1^2 + E[\tilde{b}]p_1p_2 + (E[\tilde{\phi}] - E[\tilde{a}])w - \frac{1}{2}\int_0^1(\tilde{c}_{n\alpha}^U \tilde{\phi}_\alpha^L + \tilde{c}_{n\alpha}^L \tilde{\phi}_\alpha^U)d\alpha + \\
 &E[\tilde{c}_n\tilde{a}] + E[\tilde{c}_\Delta\tilde{a}] + f(E[\tilde{a}] + p_2 - E[\tilde{b}]p_1), \tag{1}
 \end{aligned}$$

where $\tilde{c}_{n\alpha}^L = \inf\{r | Pos\{\tilde{c}_n \leq r\} \geq \alpha\}$ and $\tilde{c}_{n\alpha}^U = \sup\{r | Pos\{\tilde{c}_n \geq r\} \geq \alpha\}$ are the α -pessimistic value and the α -optimistic value of \tilde{c}_n ($0 < \alpha < 1$), respectively (Liu, 2009). Similarly, $\tilde{c}_{\Delta\alpha}^L$, $\tilde{c}_{\Delta\alpha}^U$, $\tilde{\phi}_\alpha^L$, $\tilde{\phi}_\alpha^U$, $\tilde{\beta}_\alpha^L$, $\tilde{\beta}_\alpha^U$, \tilde{a}_α^L , \tilde{a}_α^U , \tilde{b}_α^L and \tilde{b}_α^U are the α -pessimistic value and the α -optimistic value of \tilde{c}_Δ , $\tilde{\phi}$, $\tilde{\beta}$, \tilde{a} and \tilde{b} . $E[\pi_{m1}]$ is the expected profit of the manufacturer. Similarly, the expected profit of the remanufacturer can be given as follows:

$$\begin{aligned}
 E[\pi_{m2}] &= E\left[(w - \tilde{c}_n + \tilde{c}_\Delta)S_2(p_1, p_2) + (f + p_2)S_2(p_1, p_2)\right] = \\
 &= E\left[(w - \tilde{c}_n + \tilde{c}_\Delta)(\tilde{a} + p_2 - \tilde{b}p_1) - (f + p_2)(\tilde{a} + p_2 - \tilde{b}p_1)\right] = \\
 &\left(E[\tilde{c}_n\tilde{b}] - E[\tilde{b}]w - \frac{1}{2}\int_0^1(\tilde{c}_{\Delta\alpha}^U \tilde{b}_\alpha^L + \tilde{c}_{\Delta\alpha}^L \tilde{b}_\alpha^U)d\alpha\right)p_1 + (E[\tilde{c}_\Delta] - E[\tilde{c}_n] + w)p_2 + E[\tilde{a}]w + \\
 &E[\tilde{c}_\Delta\tilde{a}] - \frac{1}{2}\int_0^1(\tilde{c}_{n\alpha}^U \tilde{b}_\alpha^L + \tilde{c}_{n\alpha}^L \tilde{b}_\alpha^U)d\alpha - (f + p_2)(E[\tilde{a}] + p_2 - E[\tilde{b}]p_1). \tag{2}
 \end{aligned}$$

The retailer’s expected profit can be written as:

$$E[\pi_r] = E\left[(p_r - w)(\tilde{\phi} - \tilde{\beta}p_r)\right] = (p_r - w)(E[\tilde{\phi}] - E[\tilde{\beta}]p_r). \tag{3}$$

2. Basic models

2.1. Centralized decision-making model

In this section, we consider a C-CLSC in which the manufacturer, the remanufacturer, and the retailer make a centralized decision on the used product take-back prices p_1 , p_2 and the retail price p to maximize the expected profit of the entire CLSC system. In this model, the wholesale price w and the royalty f are non-relevant variables that affect the distribution of profits for each participant, but not affect the entire system profit.

Let $\pi_T^C(p_r, p_1, p_2)$ be the total profit of the C-CLSC, which can be given as follows:

$$\pi_T^C(p_r, p_1, p_2) = (p_r - \tilde{c}_n)(\tilde{\phi} - \tilde{\beta}p_r) + (\tilde{c}_\Delta - p_1)(a + p_1 - bp_2) + (\tilde{c}_\Delta - p_2)(a + p_2 - bp_1). \quad (4)$$

According to the above description about the model, the objective of the system in the C-CLSC is to maximize the expected profit of the entire CLSC. Using Eq. (4), the problem can be denoted as follows:

$$\text{Max}_{p_r, p_1, p_2} E[\pi_T^C] = \text{Max}_{p_r, p_1, p_2} E[(p_r - \tilde{c}_n)(\tilde{\phi} - \tilde{\beta}p_r) + (\tilde{c}_\Delta - p_1)(\tilde{a} + p_1 - \tilde{b}p_2) + (\tilde{c}_\Delta - p_2)(\tilde{a} + p_2 - \tilde{b}p_1)]. \quad (5)$$

Proposition 1. The expected profit $E[\pi_T^C]$ is concave with respect to p_r, p_1 and p_2 , the following equilibrium decisions can be obtained:

- a) The optimal retail price (denoted by p_r^C) and the optimal take-back price of the used product from the consumers (denoted by p_1^C, p_2^C respectively) are given as:

$$p_r^C = \frac{E[\tilde{\phi}] + E[\tilde{c}_n\tilde{\beta}]}{2E[\tilde{\beta}]}, \quad p_1^C = p_2^C = \frac{A}{2(1 - E[\tilde{b}])}.$$

- b) With the above optimal solution (p_r^C, p_1^C, p_2^C) , the maximum expected profit of the C-CLSC can be expressed as:

$$E[\pi_T^C] = (E[\tilde{\phi}] + E[\tilde{c}_n\tilde{\beta}])p_r^C - E[\tilde{\beta}](p_r^C)^2 + Ap_1^C + Ap_2^C + 2E[\tilde{b}]p_1^Cp_2^C + 2E[\tilde{c}_\Delta\tilde{a}] - (p_1^C)^2 - (p_2^C)^2 - \frac{1}{2} \int_0^1 (\tilde{c}_{n\alpha}^U \tilde{\phi}_\alpha^L + \tilde{c}_{n\alpha}^L \tilde{\phi}_\alpha^U) d\alpha, \quad (6)$$

where $A = E[\tilde{c}_\Delta] - E[\tilde{a}] - \frac{1}{2} \int_0^1 (\tilde{c}_{\Delta\alpha}^U \tilde{b}_\alpha^L + \tilde{c}_{\Delta\alpha}^L \tilde{b}_\alpha^U) d\alpha.$

Proof. According Assumption 1, the expected profit $E[\pi_T^C]$ can be expressed as:

$$\begin{aligned} E[\pi_T^C] &= E[(p_r - \tilde{c}_n)(\tilde{\phi} - \tilde{\beta}p_r) + (\tilde{c}_\Delta - p_1)(\tilde{a} + p_1 - \tilde{b}p_2) + (\tilde{c}_\Delta - p_2)(\tilde{a} + p_2 - \tilde{b}p_1)] = \\ & (E[\tilde{\phi}] + E[\tilde{c}_n\tilde{\beta}])p_r - E[\tilde{\beta}]p_r^2 + \left(E[\tilde{c}_\Delta] - E[\tilde{a}] - \frac{1}{2} \int_0^1 (\tilde{c}_{\Delta\alpha}^U \tilde{b}_\alpha^L + \tilde{c}_{\Delta\alpha}^L \tilde{b}_\alpha^U) d\alpha \right) p_1 + \\ & \left(E[\tilde{c}_\Delta] - E[\tilde{a}] - \frac{1}{2} \int_0^1 (\tilde{c}_{\Delta\alpha}^U \tilde{b}_\alpha^L + \tilde{c}_{\Delta\alpha}^L \tilde{b}_\alpha^U) d\alpha \right) p_2 + 2E[\tilde{b}]p_1p_2 + 2E[\tilde{c}_\Delta\tilde{a}] - \\ & p_1^2 - p_2^2 - \frac{1}{2} \int_0^1 (\tilde{c}_{n\alpha}^U \tilde{\phi}_\alpha^L + \tilde{c}_{n\alpha}^L \tilde{\phi}_\alpha^U) d\alpha. \end{aligned} \quad (7)$$

Based on Eq. (7), the first-order partial derivatives of $E[\pi_T^C(p_r, p_1, p_2)]$ with respect to p_r, p_1 and p_2 can be shown as:

$$\frac{\partial E[\pi_T^C]}{\partial p_r} = E[\tilde{\phi}] + E[\tilde{c}_n\tilde{\beta}] - 2p_rE[\tilde{\beta}]; \quad (8)$$

$$\frac{\partial E[\pi_T^C]}{\partial p_1} = E[\tilde{c}_\Delta] - E[\tilde{a}] - \frac{1}{2} \int_0^1 (\tilde{c}_{\Delta\alpha}^U \tilde{b}_\alpha^L + \tilde{c}_{\Delta\alpha}^L \tilde{b}_\alpha^U) d\alpha + 2E[\tilde{b}]p_2 - 2p_1; \quad (9)$$

$$\frac{\partial E[\pi_T^C]}{\partial p_2} = E[\tilde{c}_\Delta] - E[\tilde{a}] - \frac{1}{2} \int_0^1 (\tilde{c}_{\Delta\alpha}^U \tilde{b}_\alpha^L + \tilde{c}_{\Delta\alpha}^L \tilde{b}_\alpha^U) d\alpha + 2E[\tilde{b}]p_1 - 2p_2, \quad (10)$$

and the corresponding Hessian matrix can be expressed as follows:

$$H_1 = \begin{bmatrix} \frac{\partial^2 E[\pi_r^C]}{\partial p_r^2} & \frac{\partial^2 E[\pi_r^C]}{\partial p_r \partial p_1} & \frac{\partial^2 E[\pi_r^C]}{\partial p_r \partial p_2} \\ \frac{\partial^2 E[\pi_r^C]}{\partial p_1 \partial p_r} & \frac{\partial^2 E[\pi_r^C]}{\partial p_1^2} & \frac{\partial^2 E[\pi_r^C]}{\partial p_1 \partial p_2} \\ \frac{\partial^2 E[\pi_r^C]}{\partial p_2 \partial p_r} & \frac{\partial^2 E[\pi_r^C]}{\partial p_2 \partial p_1} & \frac{\partial^2 E[\pi_r^C]}{\partial p_2^2} \end{bmatrix} = \begin{bmatrix} -2E[\tilde{\beta}] & 0 & 0 \\ 0 & -2 & 2E[\tilde{b}] \\ 0 & 2E[\tilde{b}] & -2 \end{bmatrix}. \tag{11}$$

Based on Assumption 1, since $\begin{vmatrix} -2E[\tilde{\beta}] & 0 \\ 0 & -2 \end{vmatrix} = 4E[\tilde{\beta}] > 0$, and $H_1 = -8E[\tilde{\beta}](1 - E^2[\tilde{b}]) < 0$,

then, the Hessian matrix is negative definite. So the function $E[\pi_r^C(p_r, p_1, p_2)]$ is the joint concave function of p_r, p_1 and p_2 , and Eq. (5) has the optimal solution. Equating Eq. (8)–(10) to zero and solving them simultaneously, we can easily obtain Proposition 1 (a). Substituting Proposition 1 (a) into Eq. (7), we have Proposition 1 (b). Thus, Proposition 1 is proven.

2.2. Decentralized decision-making model

In the D-CLSC, we assume that the manufacturer is the Stackelberg leader, and the remanufacturer and the retailer are the followers. At the same time, it is assumed that the remanufacturer and the retailer act simultaneously and compete with Bertrand competition. In this case, the game process can be described as follows:

- a) The manufacturer first maximizes profit by deciding the wholesale price w , the unit take-back price p_1 and royalty f ;
- b) In response to w, p_1 and f , the remanufacturer and the retailer determine the unit take-back price p_2 and the retail price p_r based on the expected profit maximization principle.

From the above-mentioned process, we can see the game belongs to a two-phase dynamic game. The game equilibrium is a sub-game perfect Nash equilibrium and can be solved through backward induction.

Proposition 2. In the D-CLSC, considering the manufacturer’s early strategy (w, p_1, f) , the optimal response function for the remanufacturer and the retailer can be expressed as:

$$p_2^{M^*}(w, p_1, f) = \frac{E[\tilde{c}_\Delta] - E[\tilde{c}_n] - E[\tilde{a}] + E[\tilde{b}]p_1 + w - f}{2}; \tag{12}$$

$$p_r^{M^*}(w, p_1, f) = \frac{E[\tilde{\beta}]w + E[\tilde{\phi}]}{2E[\tilde{\beta}]}. \tag{13}$$

Proof. According to Eq. (2), the first-order partial derivatives of $E[\pi_{m2}]$ with respect to p_2 can be shown as:

$$\frac{\partial E[\pi_{m2}]}{\partial p_2} = E[\tilde{c}_\Delta] - E[\tilde{c}_n] + w - f - E[\tilde{a}] - 2p_2 + E[\tilde{b}]p_1. \tag{14}$$

According to Eq. (3), the first-order partial derivatives of $E[\pi_r]$ with respect to p_r can be shown as:

$$\frac{\partial E[\pi_r]}{\partial p_r} = E[\tilde{\phi}] - 2E[\tilde{\beta}]p_r + E[\tilde{\beta}]w. \tag{15}$$

We also have $\frac{\partial E[\pi_{m2}]}{\partial p_2^2} = -2E[\tilde{\beta}] < 0$ and $\frac{\partial E[\pi_r]}{\partial p_r^2} = -2 < 0$, which indicate that $E[\pi_{m2}]$ and $E[\pi_r]$ are concave in p_2 and p_r , respectively. Equating Eqs. (12)-(13) to zero and solving them, we can easily obtain Eq. (12) and Eq. (13), which completes the proof of Proposition 2.

Remark 1. In D-CLSC, the remanufacturer’s take-back price is negatively related to the unit royalty fee. That is, when the unit royalty fee increases, the remanufacturer’s take-back price will decrease. This is because when the royalty increases, the marginal cost of the remanufacturer’s production of the recycled products increases, the profit decreases, and the remanufacturer’s production enthusiasm also decreases. As a result, the remanufacturer does not have enough incentive to increase the take-back price of the used products.

After knowing the optimal response function of the remanufacturer and the retailer, the manufacturer determines the wholesale price w^{M^*} , the unit take-back price $p_1^{M^*}$ and the unit royalty fee f^{M^*} , which maximize his expected profit $E[\pi_{m1}^M]$. The following proposition has been reached.

Proposition 3. In the D-CLSC, the optimal set of strategies for the manufacturer, the remanufacturer and the retailer is $((w^{M^*}, p_1^{M^*}, f^{M^*}), p_2^{M^*}, p_r^{M^*})$. The optimal wholesale price (denoted by w^{M^*}), the optimal royalty fee (denoted by f^{M^*}), the optimal take-back prices of the used products from the consumers (denoted by $p_1^{C^*}, p_2^{C^*}$ respectively) and the retail price (denoted by $p_r^{C^*}$) are given as:

$$w^{M^*} = \frac{E[\tilde{\phi}] + E[\tilde{c}_n \tilde{\beta}]}{2E[\tilde{\beta}]};$$

$$p_1^{M^*} = \frac{E[\tilde{c}_\Delta] - E[\tilde{a}] - E[\tilde{b}] \frac{1}{2} \int_0^1 (\tilde{c}_{\Delta\alpha}^U \tilde{b}_\alpha^L + \tilde{c}_{\Delta\alpha}^L \tilde{b}_\alpha^U) d\alpha + B_1}{2(1 - E^2[\tilde{b}])};$$

$$f^{M^*} = \frac{E[\tilde{\phi}] + E[\tilde{c}_n \tilde{\beta}]}{2E[\tilde{\beta}]} + \frac{E[\tilde{a}] + E[\tilde{c}_\Delta]}{2} + \frac{E[\tilde{b}](E[\tilde{a}] - E[\tilde{c}_\Delta] - B_1) + \frac{1}{2} \int_0^1 (\tilde{c}_{\Delta\alpha}^U \tilde{b}_\alpha^L + \tilde{c}_{\Delta\alpha}^L \tilde{b}_\alpha^U)}{2(1 - E^2[\tilde{b}])} - E[\tilde{c}_n];$$

and $p_2^{M^*} = \frac{E[\tilde{c}_\Delta] - 3E[\tilde{a}] - (1 + E^2[\tilde{b}]) \frac{1}{2} \int_0^1 (\tilde{c}_{\Delta\alpha}^U \tilde{b}_\alpha^L + \tilde{c}_{\Delta\alpha}^L \tilde{b}_\alpha^U) d\alpha + B_2}{4(1 - E^2[\tilde{b}])}, p_r^{M^*} = \frac{3E[\tilde{\phi}] + E[\tilde{c}_n \tilde{\beta}]}{4E[\tilde{\beta}]},$

where

$$B_1 = E[\tilde{b}](E[\tilde{c}_n] - E[\tilde{a}]) - \frac{1}{2} \int_0^1 (\tilde{c}_{n\alpha}^U \tilde{b}_\alpha^L + \tilde{c}_{n\alpha}^L \tilde{b}_\alpha^U) d\alpha;$$

$$B_2 = 2E[\tilde{b}]\left(E[\tilde{c}_\Delta] - E[\tilde{a}] - \frac{1}{2} \int_0^1 (\tilde{c}_{n\alpha}^U \tilde{b}_\alpha^L + \tilde{c}_{n\alpha}^L \tilde{b}_\alpha^U) d\alpha\right) + E^2[\tilde{b}](2E[\tilde{c}_n] + E[\tilde{a}] - E[\tilde{c}_\Delta]).$$

Proof. According to Eq. (1), Eq. (12) and Eq. (13), the maximized expected profit function of the manufacturer is

$$\begin{aligned} & \text{Max}_{w, p_1, f} E \left[\pi_{m1}^M (w, p_1, f, p_2^{M^*} (w, p_1, f), p_r^{M^*} (w, p_1, f)) \right] = \\ & \text{Max}_{w, p_1, f} E \left[(w - \tilde{c}_n) \left[\tilde{\phi} - \tilde{\beta} p_r^{M^*} - (\tilde{a} + p_2^{M^*} - \tilde{b} p_1) \right] + (\tilde{c}_\Delta - p_1) (\tilde{a} + p_1 - \tilde{b} p_2^{M^*}) + f (\tilde{a} + p_2^{M^*} - \tilde{b} p_1) \right]. \end{aligned} \tag{16}$$

Based on Eq. (16), the first-order partial derivatives of $E[\pi_{m1}^M]$ with respect to w, p_1 and f can be shown as:

$$\frac{\partial E[\pi_{m1}^M]}{\partial w} = E[\tilde{b}] p_1 - (E[\tilde{\beta}] + 1) w + f + C_1; \tag{17}$$

$$\frac{\partial E[\pi_{m1}^M]}{\partial p_1} = E[\tilde{b}] w + (E^2[\tilde{b}] - 2) p_1 - E[\tilde{b}] f + C_2; \tag{18}$$

$$\frac{\partial E[\pi_{m1}^M]}{\partial f} = w - f - E[\tilde{b}] p_1 + C_3, \tag{19}$$

where

$$\begin{aligned} C_1 &= E[\tilde{c}_n] + \frac{1}{2} \left(E[\tilde{c}_n \tilde{\beta}] + E[\tilde{\phi}] - E[\tilde{c}_\Delta] - E[\tilde{a}] - \frac{1}{2} \int_0^1 (\tilde{c}_{\Delta\alpha}^U \tilde{b}_\alpha^L + \tilde{c}_{\Delta\alpha}^L \tilde{b}_\alpha^U) d\alpha \right); \\ C_2 &= (E[\tilde{c}_\Delta] - E[\tilde{a}]) \left(1 - \frac{1}{2} E[\tilde{b}] \right) - \frac{1}{2} \int_0^1 (\tilde{c}_{n\alpha}^U \tilde{b}_\alpha^L + \tilde{c}_{n\alpha}^L \tilde{b}_\alpha^U) d\alpha - \frac{1}{4} E[\tilde{b}] \int_0^1 (\tilde{c}_{\Delta\alpha}^U \tilde{b}_\alpha^L + \tilde{c}_{\Delta\alpha}^L \tilde{b}_\alpha^U) d\alpha; \\ C_3 &= \frac{1}{2} \left(\frac{1}{2} \int_0^1 (\tilde{c}_{\Delta\alpha}^U \tilde{b}_\alpha^L + \tilde{c}_{\Delta\alpha}^L \tilde{b}_\alpha^U) d\alpha + E[\tilde{a}] + E[\tilde{c}_\Delta] \right) - E[\tilde{c}_n], \end{aligned}$$

and the corresponding Hessian matrix can be expressed as follows:

$$H_1 = \begin{bmatrix} \frac{\partial^2 E[\pi_{m1}^M]}{\partial w^2} & \frac{\partial^2 E[\pi_{m1}^M]}{\partial w \partial p_1} & \frac{\partial^2 E[\pi_{m1}^M]}{\partial w \partial f} \\ \frac{\partial^2 E[\pi_{m1}^M]}{\partial p_1 \partial w} & \frac{\partial^2 E[\pi_{m1}^M]}{\partial p_1^2} & \frac{\partial^2 E[\pi_{m1}^M]}{\partial p_1 \partial f} \\ \frac{\partial^2 E[\pi_{m1}^M]}{\partial f \partial w} & \frac{\partial^2 E[\pi_{m1}^M]}{\partial f \partial p_1} & \frac{\partial^2 E[\pi_{m1}^M]}{\partial f^2} \end{bmatrix} = \begin{bmatrix} -E[\tilde{\beta}] - 1 & E[\tilde{b}] & 1 \\ E[\tilde{b}] & E^2[\tilde{b}] - 2 & -E[\tilde{b}] \\ 1 & -E[\tilde{b}] & -1 \end{bmatrix}. \tag{20}$$

Based on Assumption 1, since $\begin{vmatrix} -E[\tilde{\beta}] - 1 & E[\tilde{b}] \\ E[\tilde{b}] & E^2[\tilde{b}] - 2 \end{vmatrix} > 0$, and $H_1 = 2E[\tilde{\beta}] (E^2[\tilde{b}] - 1) < 0$,

then the function $E[\pi_{m1}^M (w, p_1, f, p_2^{M^*} (w, p_1, f), p_r^{M^*} (w, p_1, f))]$ is the joint concave function of w, p_1 and f , and Eq. (16) has the optimal solution. Equating Eqs (17)–(19) to zero and solving them simultaneously, we can easily obtain $w^{M^*}, p_1^{M^*}$ and f^{M^*} .

After the manufacturer determines the wholesale price w^{M^*} , the unit take-back price $p_1^{M^*}$ and the unit royalty fee f^{M^*} , the optimal unit take-back price $p_2^{M^*}$ determined by the remanufacturer and the retail price $p_r^{M^*}$ determined by the retailer can be obtained. By substituting $w^{M^*}, p_1^{M^*}$ and f^{M^*} into Eq. (12) and Eq. (13), we can easily obtain $p_2^{M^*}$ and $p_r^{M^*}$, which completes the proof of Proposition 3.

Proposition 4. According to the above optimal solution $\left((w^{M^*}, p_1^{M^*}, f^{M^*}), p_2^{M^*}, p_r^{M^*} \right)$, the optimal expected profits of the CLSC in the D-CLSC can be expressed as:

$$E[\pi_T^{M^*}] = \left(E[\tilde{c}_\Delta] - E[\tilde{a}] + E[\tilde{c}_n \tilde{b}] + E[\tilde{b}] f^{M^*} - \frac{1}{2} \int_0^1 (\tilde{c}_{n\alpha}^U \tilde{b}_\alpha^L + \tilde{c}_{n\alpha}^L \tilde{b}_\alpha^U) d\alpha - \frac{1}{2} \int_0^1 (\tilde{c}_{\Delta\alpha}^U \tilde{b}_\alpha^L + \tilde{c}_{\Delta\alpha}^L \tilde{b}_\alpha^U) d\alpha \right) p_1^{M^*} + \left(E[\tilde{c}_\Delta] - E[\tilde{a}] - f^{M^*} - \frac{1}{2} \int_0^1 (\tilde{c}_{\Delta\alpha}^U \tilde{b}_\alpha^L + \tilde{c}_{\Delta\alpha}^L \tilde{b}_\alpha^U) d\alpha \right) p_2^{M^*} + (E[\tilde{c}_n \tilde{\beta}] + E[\tilde{\phi}]) p_r^{M^*} - p_1^{M^*2} - p_2^{M^*2} - E[\tilde{\beta}] p_r^{M^*2} + 2E[\tilde{b}] p_1^{M^*} p_2^{M^*} + E[\tilde{c}_\Delta \tilde{a}] - E[\tilde{a}] f^{M^*} - \frac{1}{2} \int_0^1 (\tilde{c}_{n\alpha}^U \tilde{\phi}_\alpha^L + \tilde{c}_{n\alpha}^L \tilde{\phi}_\alpha^U) d\alpha - \frac{1}{2} \int_0^1 (\tilde{c}_{n\alpha}^U \tilde{b}_\alpha^L + \tilde{c}_{n\alpha}^L \tilde{b}_\alpha^U) d\alpha. \quad (21)$$

Proof. According to Eq. (1)–(3), the expected profit function of the CLSC is

$$E[\pi_r^M] = E[\pi_{m_1}] + E[\pi_{m_2}] + E[\pi_{m_r}] = \left(E[\tilde{c}_\Delta] - E[\tilde{a}] + E[\tilde{c}_n \tilde{b}] + E[\tilde{b}] f - \frac{1}{2} \int_0^1 (\tilde{c}_{n\alpha}^U \tilde{b}_\alpha^L + \tilde{c}_{n\alpha}^L \tilde{b}_\alpha^U) d\alpha - \frac{1}{2} \int_0^1 (\tilde{c}_{\Delta\alpha}^U \tilde{b}_\alpha^L + \tilde{c}_{\Delta\alpha}^L \tilde{b}_\alpha^U) d\alpha \right) p_1 + \left(E[\tilde{c}_\Delta] - E[\tilde{a}] - f - \frac{1}{2} \int_0^1 (\tilde{c}_{\Delta\alpha}^U \tilde{b}_\alpha^L + \tilde{c}_{\Delta\alpha}^L \tilde{b}_\alpha^U) d\alpha \right) p_2 + (E[\tilde{c}_n \tilde{\beta}] + E[\tilde{\phi}]) p_r - p_1^2 - p_2^2 - E[\tilde{\beta}] p_r^2 + 2E[\tilde{b}] p_1 p_2 + E[\tilde{c}_\Delta \tilde{a}] - E[\tilde{a}] f - \frac{1}{2} \int_0^1 (\tilde{c}_{n\alpha}^U \tilde{\phi}_\alpha^L + \tilde{c}_{n\alpha}^L \tilde{\phi}_\alpha^U) d\alpha - \frac{1}{2} \int_0^1 (\tilde{c}_{n\alpha}^U \tilde{b}_\alpha^L + \tilde{c}_{n\alpha}^L \tilde{b}_\alpha^U) d\alpha. \quad (22)$$

Then, we substitute Proposition 3 into Eq. (21), which completes the proof of Proposition 4.

3. Numerical study

The optimal strategies derived above for the CLSC are rather complex, as it involves the α -optimistic and the α -pessimistic values of the fuzzy variables. We now compare the results obtained in Section 3 through numerical study. We contrast the differences between the C-CLSC and D-CLSC, and examine how the cost-saving from the patented manufacturing technique affects the take-back prices, royalty, and profits of CLSC members.

Our research was motivated by a Chinese home appliance company. For the numerical study, we use a fictitious by representative data to show the underlying relationship of the CLSC structure. The relationship between the linguistic expressions of the uncertain parameters and the triangular fuzzy variables is typically determined by the experts' experience (Wei & Zhao, 2011, 2013). We define them in Table 2.

Assume that unite manufacturing cost \tilde{c}_n , the unit cost saving \tilde{c}_Δ , the price sensitivity coefficient $\tilde{\beta}$ are medium, and the other uncertain parameters are low or small. From Table 2, we have $\tilde{c}_n = (45, 50, 55)$, $\tilde{c}_\Delta = (15, 20, 25)$, $\tilde{\phi} = (180, 200, 220)$, $\tilde{\beta} = (0.4, 0.6, 0.8)$, $\tilde{a} = (4, 5, 6)$, $\tilde{b} = (0.2, 0.3, 0.4)$. Following Yang and Xiao (2017), we attain the expected values of these fuzzy variables as: $E[\tilde{c}_n] = 50$, $E[\tilde{c}_\Delta] = 20$, $E[\tilde{\phi}] = 200$, $E[\tilde{\beta}] = 0.6$, $E[\tilde{a}] = 5$, $E[\tilde{b}] = 0.3$, $E[\tilde{c}_n \tilde{b}] = \frac{91}{6}$, $E[\tilde{c}_n \tilde{a}] = \frac{755}{3}$, $E[\tilde{c}_n \tilde{\beta}] = \frac{91}{3}$ and $E[\tilde{c}_\Delta \tilde{a}] = \frac{305}{3}$.

The α -optimistic value and α -pessimistic value of the parameters \tilde{c}_n , \tilde{c}_Δ , \tilde{b} and $\tilde{\phi}$ are: $\tilde{c}_{n\alpha}^L = 45 + 5\alpha$, $\tilde{c}_{n\alpha}^U = 55 - 5\alpha$, $\tilde{c}_{\Delta\alpha}^L = 15 + 5\alpha$, $\tilde{c}_{\Delta\alpha}^U = 25 - 5\alpha$, $\tilde{b}_\alpha^L = 0.2 + 0.1\alpha$, $\tilde{b}_\alpha^U = 0.4 - 0.1\alpha$, $\tilde{\phi}_\alpha^L = 180 + 20\alpha$, $\tilde{\phi}_\alpha^U = 220 - 20\alpha$. Thus, we have $\frac{1}{2} \int_0^1 (\tilde{c}_{n\alpha}^U \tilde{b}_\alpha^L + \tilde{c}_{n\alpha}^L \tilde{b}_\alpha^U) d\alpha = \frac{89}{6}$, $\frac{1}{2} \int_0^1 (\tilde{c}_{\Delta\alpha}^U \tilde{b}_\alpha^L + \tilde{c}_{\Delta\alpha}^L \tilde{b}_\alpha^U) d\alpha = \frac{35}{6}$, $\frac{1}{2} \int_0^1 (\tilde{c}_{n\alpha}^U \tilde{\phi}_\alpha^L + \tilde{c}_{n\alpha}^L \tilde{\phi}_\alpha^U) d\alpha = \frac{29900}{3}$.

Table 2. Relationship between linguistic expression and triangular fuzzy variable

	Linguistic expression	Fuzzy variable
Unit manufacturing cost \tilde{c}_n	High (about 60)	(55, 60, 65)
	Medium (about 50)	(45, 50, 55)
	Low (about 40)	(35, 40, 45)
Unit cost saving \tilde{c}_A	High (about 30)	(25, 30, 35)
	Medium (about 20)	(15, 20, 25)
	Low (about 10)	(5, 10, 15)
Market capacity $\tilde{\phi}$	Large (about 400)	(380, 400, 420)
	Medium (about 300)	(280, 300, 320)
	Small (about 200)	(180, 200, 220)
Price sensitivity coefficient $\tilde{\beta}$	High (about 0.8)	(0.4, 0.6, 0.8)
	Medium (about 0.6)	(0.4, 0.6, 0.8)
	Low (about 0.4)	(0.2, 0.4, 0.6)
Market scale \tilde{a}	Large (about 9)	(8, 9, 10)
	Medium (about 7)	(6, 7, 8)
	Small (about 5)	(4, 5, 6)
Price elasticity \tilde{b}	High (about 0.8)	(0.7, 0.8, 0.9)
	Medium (about 0.5)	(0.4, 0.5, 0.6)
	Low (about 0.3)	(0.2, 0.3, 0.4)

From the theoretical results derived in Section 3, we can compute the optimal wholesale prices, retail prices, take-back prices and maximal expected profits for C-CLSC and D-CLSC respectively. The corresponding results are shown in Tables 3 and 4, respectively.

From Tables 3 and 4, we find

- 1) The optimal retail price in C-CLSC is lower than that in D-CLSC. Lower retail price in C-CLSC can benefit consumers and lead to more sales.
- 2) The take-back prices in C-CLSC is no less than that in D-CLSC. Higher take-back prices will increase the return volume of the used products.
- 3) The total profit in C-CLSC is higher than that in D-CLSC.

Table 3. Optimal values of the decision variables

Scenario	P_r^*	P_1^*	P_2^*	w^*	f^*
Centralized decision	191.94	6.55	6.55		
Decentralized decision	262.64	6.55	1.76	191.94	155.40

Table 4. Optimal expected profits of entire system and each individual firm

Scenario	$E[\pi_T^*]$	$E[\pi_{m_1}]$	$E[\pi_{m_2}]$	$E[\pi_{m_r}]$
Centralized decision	12402.30			
Decentralized decision	9619.71	6359.10	262.97	2998.64

Overall, we find C-CLSC dominates D-CLSC in terms of the consumer surplus (high take-back price, low retail price), CLSC profit (increase sales) and the society wellbeing (improve resource utilization and environmental protection).

We also study how cost-saving intensity affects channel players' pricing strategy, the expected profits, and the manufacturer's royalty fee on remanufacturer. Assuming that the expected value of the fuzzy variable \tilde{c}_Δ increases from 20 to 35, holding the fuzzy degree and other parameters constant, then we can obtain the optimal solutions and the optimal expected profits, as shown in Tables 5 and 6.

Table 5. Centralized decision-making with the change of expected value of \tilde{c}_Δ

$E[\tilde{c}_\Delta]$	$p_r^{C^*}$	$p_1^{C^*}$	$p_2^{C^*}$	$E[\pi_T^{C^*}]$
20	191.9	6.55	6.55	12402.30
25	"	9.05	9.05	12506.90
30	"	11.55	11.55	12629.00
35	"	14.05	14.05	12768.50

Table 6. Decentralized decision-making with the change of expected value of \tilde{c}_Δ

$E[\tilde{c}_\Delta]$	w^{M^*}	$p_r^{M^*}$	f^{M^*}	$p_1^{M^*}$	$p_2^{M^*}$	$E[\pi_{m_1}^{M^*}]$	$E[\pi_{m_2}^{M^*}]$	$E[\pi_{m_r}^{M^*}]$	$E[\pi_T^{M^*}]$
20	191.94	262.64	155.40	6.55	1.76	6359.10	261.97	2998.64	9619.71
25	"	"	157.90	9.05	3.38	6445.38	271.96	"	9715.98
30	"	"	160.40	11.55	5.01	6546.10	283.47	"	9828.21
35	"	"	162.90	14.05	6.63	6636.26	296.52	"	9931.42

From Tables 5 and 6, we find

- 1) Retail price is independent of the unit cost-saving amount \tilde{c}_Δ regardless of the SC structure. The wholesale price and the retailer's expected profit of D-CLSC is also independent of \tilde{c}_Δ . The retail price in C-CLSC equals the wholesale price in D-CLSC. Thus, C-CLSC will have higher sales volume and lower consumer cost.
- 2) In both C-CLSC and D-CLSC, the take-back price for the two manufacturers and the total profits of the CLSC increase with \tilde{c}_Δ . The profits of the two manufacturers and the royalty also increase with \tilde{c}_Δ . As the unit cost-saving increases, remanufacturing cost decreases and the profit increases. Both the manufacturer and the remanufacturer are motivated to increase their take-back prices to collect more used products, thereby gaining more profits through remanufacturing. Besides, when the remanufacturer gains more profits because of the reduction in remanufacturing cost, the manufacturer would like to gain more profits by raising the royalty fee. Therefore, the unit royalty fee increases with the increase of \tilde{c}_Δ , which is consistent with the actual situation.

4. Comparative analyses and discussions

From the above analysis, we find the C-CLSC has higher operational efficiency and can avoid the loss due to double marginalization effect. However, C-CLSC is often difficult to achieve in practice, as CLSC members often focus on their own profit maximization. In the meantime, information asymmetry, uncertain new product demands, and uncertain used product supply could all affect CLSC members’ decisions. We thus further analyze how asymmetric information and uncertain markets impact channel player’s behaviors and the profits of the channels.

4.1. Impact of information asymmetry

In D-CLSC, the manufacturer could use his information advantage to hide the true manufacturing cost information from other CLSC members. We study the effect of manufacturing-cost information asymmetry on CLSC members’ decisions and profits. We keep the expected value of the fuzzy variable \tilde{c}_n unchanged at 50. Let (Δ_1, Δ_2) increase from (5, 5) to (25, 25), and $\Delta_1 + \Delta_2$ be the fuzzy degree of parameter \tilde{c}_n . The higher the fuzzy degree \tilde{c}_n , the greater the uncertainty of information asymmetry. Thus, we can derive the optimal solution for each decision variable and the optimal profits when $\tilde{c}_n = (50 - \Delta_1, 50, 50 + \Delta_2)$ (see Table 7).

Table 7. Optimal values of the decision variables and the expected profits

(Δ_1, Δ_2)	w^{M^*}	$p_r^{M^*}$	f^{M^*}	$p_1^{M^*}$	$p_2^{M^*}$	$E[\pi_{m_1}^{M^*}]$	$E[\pi_{m_2}^{M^*}]$	$E[\pi_r^{M^*}]$	$E[\pi_T^{M^*}]$
(5, 5)	191.94	262.64	155.40	6.55	1.76	6359.10	261.97	2998.64	9619.71
(10, 10)	192.22	262.78	155.65	6.64	1.78	6482.76	263.28	2986.86	9732.90
(15, 15)	192.50	262.92	155.90	6.73	1.81	6606.49	264.62	2975.10	9846.21
(20, 20)	192.78	263.06	156.15	6.82	1.84	6730.28	265.98	2963.37	9959.63
(25, 25)	193.06	263.19	156.40	6.91	1.87	6981.55	267.38	2951.66	10200.60

From Table 7, we can find the impacts of information asymmetry on various CLSC performances as shown in Figures 2–11, from which we find:

- 1) The optimal wholesale price w^{M^*} and the optimal retail price $p_r^{M^*}$ of new product will increase as the uncertainty of information asymmetry increases (Figures 2–3). This is consistent with our intuition, as the uncertainty of asymmetric information increases, the manufacturer uses his own information advantage to increase profit by increasing the wholesale price of new products. As a result, it leads to an increase in retail price, which ultimately reduce market demand for new products.
- 2) The difference between the optimal retail price and the optimal wholesale price $p_r^{M^*} - w^{M^*}$ will decrease with the information asymmetry uncertainty (Figures 4–5), which ultimately leads to the decline of the retailer’s profit $E[\pi_r^{M^*}]$.
- 3) The manufacturer and remanufacturer’s unit take-back prices $p_1^{M^*}, p_2^{M^*}$ as well as royalty f^{M^*} will increase as the uncertainty of information asymmetry increases (Figures 6–8). This is because, as the uncertainty of asymmetric information increases, the

profits that the manufacturer and the remanufacturer receive from remanufacturing increase. Thus, the manufacturer and the remanufacturer will be more motivated to increase the recycling price of the used products to obtain more waste products, so as to obtain more benefits from remanufacturing. At the same time, as the remanufacturer's profitability increases, the manufacturer increases his unit royalty fee to obtain more profits. However, the remanufacturer is willing to pay higher royalty fee to the manufacturer because of his higher profitability.

- 4) The expected profits of the manufacturer, the remanufacturer as well as the entire CLSC will increase as the uncertainty of information asymmetry increases (see Figures 9–11). However, we can notice that the manufacturer's expected profit increases more quickly than that of the remanufacturer. That is, the level of asymmetric information uncertainty has more impact on manufacturer's profit. This is because in the case of information asymmetry, the manufacturer exploits his information advantage to benefit from CLSC. Although the expected profit of the CLSC does not decline, it is not conducive to the long-term collaboration between CLSC members.

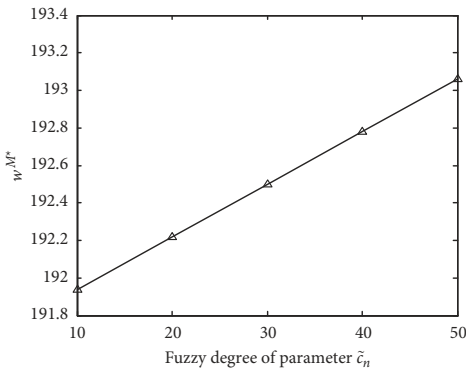


Figure 2. Information asymmetry's impact on the w_r^{M*}

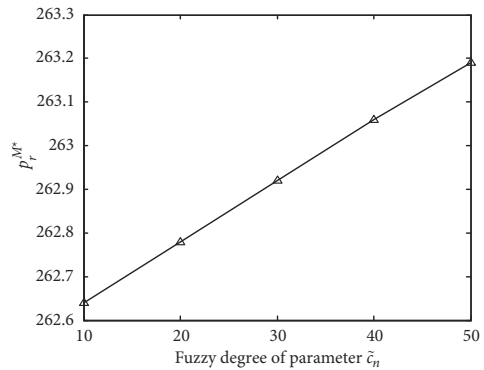


Figure 3. Information asymmetry's impact on the p_r^{M*}

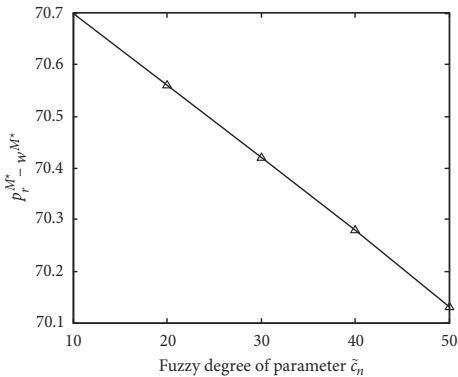


Figure 4. Information asymmetry's impact on the $p_r^{M*} - w_r^{M*}$

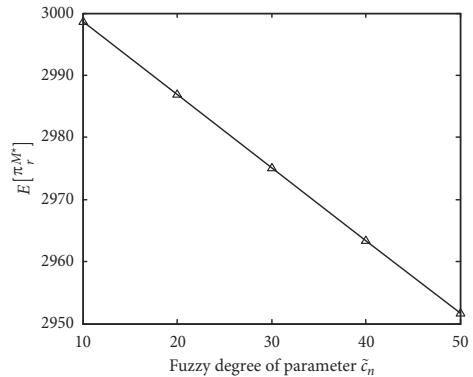


Figure 5. Information asymmetry's impact on the $E[\pi_r^{M*}]$

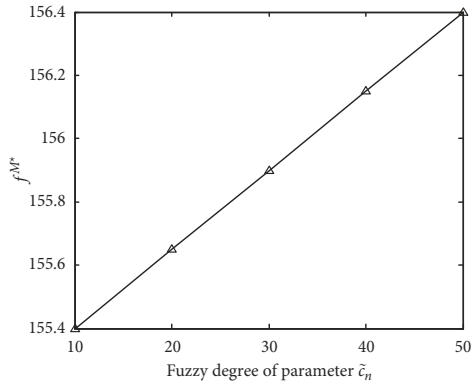


Figure 6. Information asymmetry's impact on the f^{M^*}

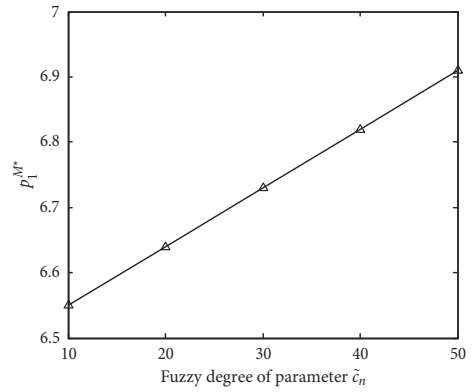


Figure 7. Information asymmetry's impact on the $p_1^{M^*}$

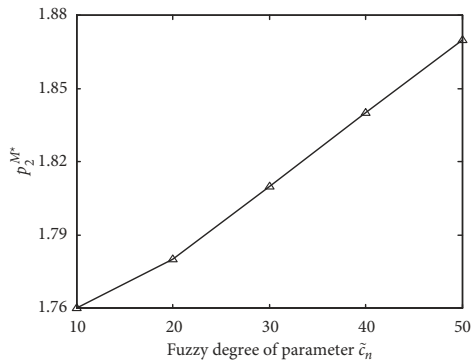


Figure 8. Information asymmetry's impact on the $p_2^{M^*}$

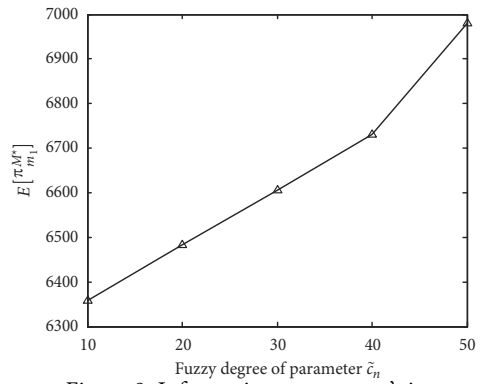


Figure 9. Information asymmetry's impact on the $E[\pi_{m_1}^{M^*}]$

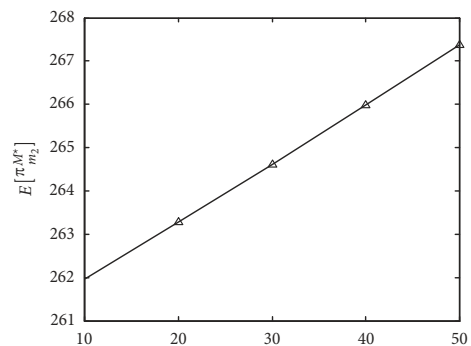


Figure 10. Information asymmetry's impact on the $E[\pi_{m_2}^{M^*}]$

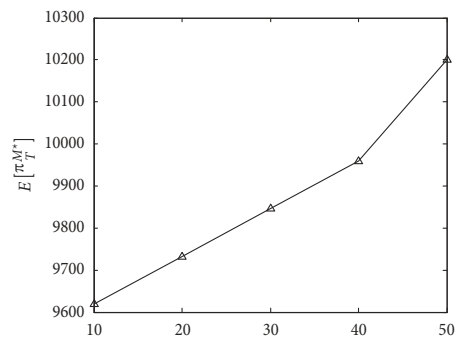


Figure 11. Information asymmetry's impact on the $E[\pi_T^{M^*}]$

4.2. Impact of market uncertainty

We now consider the impact of demand uncertainty on profits under D-CLSC. The expected value of the fuzzy variable $\tilde{\phi}$ remains unchanged at 200. Let (Δ_3', Δ_4') increase from (20, 20) to (120, 120), and $\Delta_3' + \Delta_4'$ be the fuzzy degree of the parameter $\tilde{\phi}$. The higher the fuzzy degree of $\tilde{\phi}$, the greater the uncertainty of the market demand. We can then obtain the optimal expected profits for various levels of $\tilde{\phi} = (200 + \Delta_3', 200, 200 - \Delta_4')$, as shown in Table 8.

Table 8. Optimal values of the decision variables

(Δ_3', Δ_4')	$E[\pi_{m_1}^{M^*}]$	$E[\pi_{m_2}^{M^*}]$	$E[\pi_r^{M^*}]$	$E[\pi_T^{M^*}]$
(20, 20)	6359.10	261.97	2998.64	9619.71
(50, 50)	6409.10	“	“	9669.71
(70, 70)	6442.43	“	“	9703.04
(100, 100)	6492.43	“	“	9753.04
(120, 120)	6525.76	“	“	9786.37

From Table 8, we find that as the market demand uncertainty increases, the manufacturer’s profit $E[\pi_{m_1}^{M^*}]$ also increases. This is because in the D-CLSC Stackelberg game, the manufacturer is the leader and has more market information, thus can gain more from the CLSC. We also find that the total profit $E[\pi_T^{M^*}]$ of the CLSC does not decline with the growth of demand uncertainty. However, it is worth noting that it requires profit sharing strategy to maintain the long-term cooperation and collaboration between the all members of CLSC.

Finally, we consider the case of the used products market return uncertainty. In such a case, we keep the expected value of the fuzzy variable \tilde{a} unchanged at 5. Let (Δ_5', Δ_6') increase from (1, 1) to (4, 4), and $\Delta_5' + \Delta_6'$ be the fuzzy degree of the parameter \tilde{a} . The higher the fuzzy degree of \tilde{a} , the greater the uncertainty of the market supply. Thus, we can obtain the optimal expected profits when $\tilde{a} = (5 - \Delta_5', 5, 5 + \Delta_6')$, as shown in Table 9.

Table 9. Optimal values of the decision variables

(Δ_3', Δ_4')	$E[\pi_{m_1}^{M^*}]$	$E[\pi_{m_2}^{M^*}]$	$E[\pi_r^{M^*}]$	$E[\pi_T^{M^*}]$
(1, 1)	6359.10	261.97	2998.64	9619.71
(2, 2)	6362.43	263.64	“	9624.71
(3, 3)	6365.76	265.31	“	9629.71
(4, 4)	6369.10	266.97	“	9634.71

Table 9 shows that when the uncertainty of market supply of the used products increases, the expected profits of the manufacturers and the remanufacturers also increase. High uncertainty entails greater opportunity to gain, thus the larger the possibility of obtaining higher profit. For the remanufacturing, the trend is not obvious.

Concluding remarks

In this paper, we have studied a dual-channel recycling CLSC model with patented licensing and explored operational settings to maximize the expected profits of the channels. Numerical examples are given to demonstrate how the cost-reducing technique will affect the channel players' behaviors and CLSC's royalty strategy. Through research, we have found that patented technique royalty is an effective way to resolve technical or economic infeasibility in product remanufacturing. By using the advanced production or management techniques of other companies to improve the production or management level of the enterprise, it ensures that the remanufacturing is more technically or economically feasible. Thus, the licensors and licensees of the technology can mutually benefit. Based on the theoretical derivation and the numerical examples, we obtain the following conclusions: (1) Regardless of C-CLSC or D-CLSC, the take-back price of the two manufactures and the total profits of the CLSC system will rise with the cost-saving from remanufacturing. (2) Under D-CLSC, the profits of the two manufacturers and the royalty fee also increase with the cost-saving from remanufacturing. (3) When the information asymmetry increases, the wholesale price, the retail price, the take-back prices, as well as the royalty fee will rise in D-CLSC, but retailer's expected profit will decline. (4) In the D-CLSC, the expected profits of the manufacturer will rise with market uncertainty. For the remanufacturer, this trend is not obvious.

Our main contribution is developing a direct dual-channel recycling CLSC model and investigating the cost-reducing technique royalty strategy for the patented products while competing in both the sales and recycling markets. We have explored how the cost-saving will affect the channel player's behaviors and the patented royalty strategy of the CLSC. The information asymmetry and the market uncertainty have also been considered in the D-CLSC. We also find that as the cost-reducing technique saves more remanufacturing costs, channel players have more incentives to participate in recycling of the used products. Patented product manufactures can also share more profits by increasing the royalty fee. As a result, the expected profit of the entire CLSC system has also increased. In addition, the asymmetric information and uncertainty markets have a greater impact on the manufacture than on the other channel players. The findings of this paper offer theoretical and methodological guidance for the implementation of patented technique royalty between manufacturing and remanufacturing companies, and can also provide evidence for relevant government departments to monitor and evaluate the implementation of remanufacturing companies in product recycling and remanufacturing.

Future research can be expanded in two directions: (1) Study the impact of bargaining power between patent licensors and licensees on unit royalty and CLSC decision-making. This article has only studied the situation where the licensee does not have the bargaining power. The issue that the licensee has the ability to bargain is worthy of further study. (2) Study different pricing for new products and remanufactured products. This article has assumed that the prices of new products and remanufactured products are the same, and consumers have the same degree of acceptance of both. In future studies, the impact of different pricings on the CLSC decisions can be considered.

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