



SELECTING TARGET MARKET BY SIMILAR MEASURES IN INTERVAL INTUITIONISTIC FUZZY SET

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Abstract. The selection of the target market plays vital role in promoting the marketing strategies of companies. We presented is a method for target market selection. We introduce some novel similarity measures between intuitionistic fuzzy sets and the novel similarity measures between interval-valued intuitionistic fuzzy sets. They are constructed by combining exponential and other functions. Finally, we introduce a multi-criteria decision making model to select target market by using the novel similarity measure of interval intuitionistic fuzzy sets.

Keywords: intuitionistic fuzzy set, interval – valued intuitionistic fuzzy set, similarity measure, target market, market segment.

JEL Classification: C02, C44, D40, D81.

Introduction

The target market consists of a set of customer groups having common needs or characteristics that the organizations wish to serve. The selecting of target market helps the suppliers choosing which and how many segments to serve (Bernstein, 2014). Businesses understand that customers are diverse in demands, attitudes and preferences. There is no organization which can satisfy all customers in the markets. Instead of competing in the marketplace, trying to fight against competitors, they should identify the most attractive segment that they can provide effectively (Bernstein, 2014; Kotler & Armstrong, 2003). Marketing strategies allow organizations to find out and develop markets or tailor their services to meet the needs of potential markets. There are normally three ways to reach the market: mass marketing, target marketing and consumer marketing. Mass marketing, which assumes that all customers are treated similarly, will no longer be effective in the coming century (Kuo, Ho & Hu, 2002). Instead, it will be a targeted marketing strategy, which identifies target segments and

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marketing plans that fit into it (Kotler & Armstrong, 2003). Market segmentation also raises profit or effectiveness of firms (Chiu, Chen, & Kuo, 2009). Besides that, it helps businesses to recognize the position of their product on the market, explore new opportunities and gain competitive advantages through differentiation of the product (Kotler, 1980). Any segmentation scheme should have the following characteristics: exhaustiveness, reliability and operability. By analyzing the attractiveness of each segment, managers make strategic decisions about their sources and finance for each segment, deciding on quality of the product, price and finally determining the time introduce goods to the market (Bernstein, 2014).

There are a lot of methods to segment the market but not all segments are effective. Kotler and Armstrong (2003) pointed out an example of the salt market to prove this claim. It is clear that race or ethical disposition does not affect the amount of salt consumption, so this segment is not effective. According to the authors, the market segment should have the following criteria: measurable, accessible, substantial and actionable. In Simkin and Dibb's view (1998), target marketing plays a key role in making a decision for firms. Although the activities of marketers are changing on the market, selecting the target market is still an important strategic issue. It is necessary to study the market attractiveness criteria. Market size, exchange rate growth, competitive forces, appropriate customers and profits are just a few of the many criteria that can be applied to assess the attractiveness of a target market of an enterprise. The study also found that many firms continue to evaluate target markets on the basis of short-term profit-taking measures. The target market is the center of the marketing decisions, which are seen as the output of evaluation and selection the market segmentation (Aghdaie, 2015). Selecting the target segment is a complex process that takes into account many factors such as the segment's expected response to marketing variables, the likelihood of success, the competitive nature of the segment management resources and the ability to deploy segmentation strategies (Winter, 1979). Thus, the target market selection can be considered as a multi-criteria decision making model.

A well-known and useful method for selecting the most suitable alternative from a set of alternatives is the multi-criteria decision making (MCDM). It has been applied widely to the complexity of economic decision problems as well as many other fields. In these cases, it is difficult for the makers to show their opinions by crisp numbers, so the linguistic variables is used efficiently. The linguistic variables have been recognized as a useful approach in decision making problems. In 1986, Atanassov generalized Zadeh's fuzzy set (Xu, 2010) by introducing the intuitionistic fuzzy set (IFS) (Atanassov, 1986), which considers the information for both the membership function and non-membership function. From its inception to the present day, IFS proved to be very effective in dealing with the uncertainty in the real-world problems such as the pattern recognition, the decision making, etc. After that, the interval-valued intuitionistic fuzzy set (IVIFS) was introduced by Atanassov and Gargov (1989), in which the membership function and non-membership function are subintervals of $[0, 1]$. As opposed to fuzzy sets, intuitionistic fuzzy (IF) set also has broad applications for uncertain data processing such as decision making, medical diagnosis, agriculture, image segmentation (Buhaesku, 1988; Hung & Wu, 2002; Mitchell, 2004; Szmidski & Kacprzyk, 2004; Ye, 2011; Park, Hwang, Park, Wei & Lee, 2013; Shi & Ye, 2013; Shidpour, Bernard & Shahrokhi, 2013; Bharati & Singh, 2014; Li & Zeng, 2015; Liu, Shen, Mu, Chen & Chen, 2016; Zhou, Zhao, Yu & Tian, 2016). Along with distance measurements, correlation measurements, similar

measurement of IF sets and interval IF sets is also studied and widely used in many areas and now it still is a hot topic (Chiang & Lin, 1999; Gerstenkorn & Mańko, 1991; Ye, 2016). Similarity measures are tools to show the similarity of two objects. It is useful to handle the problems such as decision making, machine learning, pattern recognition (Szmidski & Kacprzyk, 1996; Liang & Shi, 2003; Hwang, Yang, Hung & Lee, 2012; Li & Cheng, 2002; Xu, 2007b; Zeng & Wang, 2011; Ye, 2011; Park, Hwang, Park, Wei & Lee, 2013). The similarity measure (SM) between IF sets were introduced by Liang and Shi (2003) and applied in the pattern recognition problem. Xu (2006) developed the similarity measure which constructed by the distance measure in multi-criteria decision making. This problem was proposed and applied in the multi-criteria decision making by Zhou (2016). The proposed methods used polynomial functions or fractional functions. Ye (2016) generalized new version of weighted cosine similarity measures between two IF sets through the degrees of membership, non-membership and hesitancy in IF sets and applied to mechanical decision schemes using intuitionistic fuzzy numbers. Phong and Son (2017) resolved linguistic classification problem by the linguistic SM, the linguistic vector, and the linguistic vector similarity.

The existing models which evaluate and select market segment or target market, usually use the triangular or trapezoidal fuzzy numbers to express the assessment of decision makers (DMs) (see for example Aghdaie, Zolfani & Zavadskas, 2013). The interval-valued intuitionistic fuzzy set is a stronger tool in reflecting the vague features of the subjective judgments of the DMs. The proposed similarity measures of Shi and Ye (2013), Ye (2016) have been successfully applied in classification or pattern recognition, however, they were not effective in some cases. These inspire us to introduce a new similarity measure of intuitionistic fuzzy sets which can overcome some drawbacks and apply the new measure for selecting target market. The rest of this paper is as follows. In Section 1, we recall the concept of intuitionistic fuzzy set, interval-valued intuitionistic fuzzy set and the similarity measure of them. The new similarity measure between the IFs were built by combining exponential and other functions in Section 2 and we will compare the results computed by our method to other ones. The similarity measure of the interval-valued intuitionistic fuzzy sets is proposed in Section 3. In Section 4, we use the similarity measure of interval-valued intuitionistic fuzzy sets in multi-criteria decision making model. Section 5, a case study will carry out to apply the new measure for target market selection. Finally, we compare the proposed method with some existing models.

1. Preliminary

Let X be a universal set. We recall the concepts of IFS and interval-valued intuitionistic fuzzy set (IVIFSs) on X .

Definition 1. (Atanassov, 1986) An IFS X is defined by form

$$A = \left\{ \left(x, \mu_A(x), \nu_A(x) \right) \mid x \in X \right\},$$

in which $\mu_A(x) \in [0,1]$ is the membership degree and $\nu_A(x) \in [0,1]$ is the non-membership of the element x in X to A , respectively such as

$$\mu_A(x) + \nu_A(x) \leq 1, \forall x \in X.$$

For two IF sets $A, B \in IFS(X)$ we have:

- $A \subset B$ if only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$.
- $A = B$ if only if $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$ for all $x \in X$.
- The complement of intuitionistic fuzzy set A is

$$A^C = \left\{ \left(x, \nu_A(x), \mu_A(x) \right) \mid x \in X \right\}.$$

Definition 2. (Atanassov, 1989) An interval – value IFS on X is a defined by form

$$A = \left\{ \left(x, \left[\mu_A^L(x), \mu_A^U(x) \right], \left[\nu_A^L(x), \nu_A^U(x) \right] \right) \mid x \in X \right\},$$

in which $\left[\mu_A^L(x), \mu_A^U(x) \right] \subseteq [0,1]$ and $\left[\nu_A^L(x), \nu_A^U(x) \right] \subseteq [0,1]$ are the membership degree and the non-membership of the element x in X to A , respectively, and

$$\mu_A^U(x) + \nu_A^U(x) \leq 1, \forall x \in X.$$

Let $A, B \in IVIFS(X)$, we have some following characteristics (Xu, 2007a):

- $A \subset B$ if only if $\mu_A^L(x) \leq \mu_B^L(x)$, $\mu_A^U(x) \leq \mu_B^U(x)$ and $\nu_A^L(x) \geq \nu_B^L(x)$, $\nu_A^U(x) \geq \nu_B^U(x)$ for all $x \in X$.
- $A = B$ if only if $\mu_A^L(x) = \mu_B^L(x)$, $\mu_A^U(x) = \mu_B^U(x)$ and $\nu_A^L(x) = \nu_B^L(x)$, $\nu_A^U(x) = \nu_B^U(x)$ for all $x \in X$.
- Complement of the interval – value IFS A is

$$A^c = \left\{ \left(x, \left[\nu_A^L(x), \nu_A^U(x) \right], \left[\mu_A^L(x), \mu_A^U(x) \right] \right) \mid x \in X \right\}.$$

For convenience, we denote $x = \left(\left[\mu^L, \mu^U \right], \left[\nu^L, \nu^U \right] \right)$ with $\left[\mu^L, \mu^U \right] \subseteq [0,1]$, $\left[\nu^L, \nu^U \right] \subseteq [0,1]$ and $\nu^U + \mu^U \leq 1$ to show the value in IVIFs, and call it the interval intuitionistic fuzzy value.

Definition 3. (Xu, 2007a). Let x and y be two interval intuitionistic fuzzy values. We have the following basic operations:

$$\begin{aligned} x \oplus y &= \left(\left[\mu_1^L + \mu_2^L - \mu_1^L \mu_2^L, \mu_1^U + \mu_2^U - \mu_1^U \mu_2^U \right], \left[\nu_1^L \nu_2^L, \nu_1^U \nu_2^U \right] \right); \\ x \otimes y &= \left(\left[\mu_1^L \mu_2^L, \mu_1^U \mu_2^U \right], \left[\nu_1^L + \nu_2^L - \nu_1^L \nu_2^L, \nu_1^U + \nu_2^U - \nu_1^U \nu_2^U \right] \right); \\ nx &= \left(\left[1 - (1 - \mu_1^L)^n, 1 - (1 - \mu_1^U)^n \right], \left[(\nu_1^L)^n, (\nu_1^U)^n \right] \right), n > 0. \end{aligned}$$

Now, we recall the similarity measure of the intuitionistic fuzzy sets. Given $X = \{x_1, x_2, \dots, x_n\}$ is a universal set. Let $A = \left\{ \left(x_i, \mu_A(x_i), \nu_A(x_i) \right) \mid x_i \in X \right\}$, $B = \left\{ \left(x_i, \mu_B(x_i), \nu_B(x_i) \right) \mid x_i \in X \right\}$ be two IF sets on X

Definition 4. (Szmidt & Kacprzyk, 2004) A mapping $S: IFS(X) \times IFS(X) \rightarrow [0,1]$ is a SM of the intuitionistic fuzzy sets if the following conditions are satisfied:

- (i) $0 \leq S(A, B) \leq 1, \forall A, B \in IFS(X)$;
- (ii) $S(A, B) = S(B, A), \forall A, B \in IFS(X)$;
- (iii) $S(A, A) = 1, \forall A \in IFS(X)$.
- (iv) For all $A, B, C \in IFS(X)$ such that $A \subseteq B \subseteq C$ we have $S(A, C) \leq \min \{S(A, B), S(B, C)\}$.

2. New similarity measure of the IFSs

We recall the concept of Hausdorff distance which is important and effective in pattern recognition and classifying problem. For any two intervals $A=[a_1, a_2], B=[b_1, b_2]$ in the real space, the Hausdorff distance is defined by $H(A, B) = \max\{|a_1 - b_1|, |a_2 - b_2|\}$ (Nadler, 1978; Hung & Yang, 2004).

The distance and the similarity measure are dual couple. The similarity measure can define via the distance function (Hung & Yang, 2004). The exponential function has been acknowledged as a useful tool to measure the similarity (Zadeh, 1971; N. R. Pal & S. K. Pal, 1992). Therefore, motivate by the exponential function and the Hausdorff distance, in this section, we will propose a new similarity which combines them. We also compare our measure with some other ones.

Let $X = \{x_1, x_2, \dots, x_n\}$ be a finite set. For $A, B \in IFS(X)$ we denote $S_i^H(A, B) = e^{-|\mu_A(x_i) - \mu_B(x_i)|}$, $S_i^V(A, B) = 1 - |v_A(x_i) - v_B(x_i)|$ ($i = 1, 2, \dots, n$).

Definition 5. We define a mapping $S_0 : IFS(X) \times IFS(X) \rightarrow [0, 1]$ by

$$S_0(A, B) = \frac{1}{n} \sum_{i=1}^n S_i^H(A, B) \times S_i^V(A, B). \tag{1}$$

Theorem 1. The function $S_0(A, B)$ in Eq. (1) is a new similarity measure on $IFS(X)$.

Proof.

(i) We have $0 \leq S_i^H(A, B) = e^{-|\mu_A(x_i) - \mu_B(x_i)|} \leq 1$, $0 \leq S_i^V(A, B) = 1 - |v_A(x_i) - v_B(x_i)| \leq 1$ for all $A, B \in IFS(X)$. So that

$$0 \leq S_0(A, B) = \frac{1}{n} \sum_{i=1}^n S_i^H(A, B) \times S_i^V(A, B) \leq 1 \text{ for all } A, B \in IFS(X).$$

(ii) It is obvious.

(iii) For all $A, B \in IFS(X)$. If $A = B$ then $\mu_A(x_i) = \mu_B(x_i)$, $v_A(x_i) = v_B(x_i)$. We have $S_i^H(A, B) = 1$, $S_i^V(A, B) = 1$ and $S_0(A, B) = \frac{1}{n} \sum_{i=1}^n S_i^H(A, B) \times S_i^V(A, B) = 1$.

(iv) For all $A, B, C \in IFS(X)$ such that $A \subseteq B \subseteq C$ then $\mu_A(x_i) \leq \mu_B(x_i) \leq \mu_C(x_i)$ and $v_C(x_i) \leq v_B(x_i) \leq v_A(x_i)$. This implies

$$\max\{|\mu_A(x_i) - \mu_B(x_i)|, |\mu_B(x_i) - \mu_C(x_i)|\} \leq |\mu_A(x_i) - \mu_C(x_i)|$$

and

$$\max\{|v_A(x_i) - v_B(x_i)|, |v_B(x_i) - v_C(x_i)|\} \leq |v_A(x_i) - v_C(x_i)|.$$

Then we have

$$\min\{-|\mu_A(x_i) - \mu_B(x_i)|, -|\mu_B(x_i) - \mu_C(x_i)|\} \geq -|\mu_A(x_i) - \mu_C(x_i)|$$

and

$$\min\{-|v_A(x_i) - v_B(x_i)|, -|v_B(x_i) - v_C(x_i)|\} \geq -|v_A(x_i) - v_C(x_i)|.$$

Hence,

$$S_i^H(A, C) \leq \min\{S_i^H(A, B), S_i^H(B, C)\} \text{ and } S_i^V(A, C) \leq \min\{S_i^V(A, B), S_i^V(B, C)\}.$$

Thus

$$S_0(A, C) \leq \min\{S_0(A, B), S_0(B, C)\}. \square$$

Now, we can define the similarity measure of the IF sets that assigned with the weight of each element in the universal set.

We assume that each element x_i in the universal $X = \{x_1, x_2, \dots, x_n\}$ is assigned with a weight $\omega_i \in [0, 1]$ ($i = 1, 2, \dots, n$) such that $\sum_{i=1}^n \omega_i = 1$.

Definition 6. We define a mapping $S_0^\omega : IFS(X) \times IFS(X) \rightarrow [0, 1]$ by

$$S_0^\omega(A, B) = \sum_{i=1}^n \omega_i S_i^H(A, B) \times S_i^V(A, B) \tag{2}$$

Theorem 2. The function $S_0^\omega(A, B)$ in Eq. (2) is a new similarity measure on $IFS(X)$.

Proof.

(i) Because $0 \leq S_i^H(A, B), S_i^V(A, B) \leq 1$, we have

$$0 \leq S_0^\omega(A, B) = \sum_{i=1}^n \omega_i S_i^H(A, B) \times S_i^V(A, B) \leq \sum_{i=1}^n \omega_i = 1.$$

(ii) It is obvious.

(iii) If $A = B$ then $\mu_A(x_i) = \mu_B(x_i), \nu_A(x_i) = \nu_B(x_i)$. We have $S_i^H(A, B) = 1, S_i^V(A, B) = 1$.

$$\text{So that } S_0^\omega(A, B) = \sum_{i=1}^n \omega_i S_i^H(A, B) \times S_i^V(A, B) = \sum_{i=1}^n \omega_i = 1.$$

(iv) For all $A, B, C \in IFS(X)$ such that $A \subseteq B \subseteq C$

$$S_i^H(A, C) \leq \min\{S_i^H(A, B), S_i^H(B, C)\} \text{ and } S_i^V(A, C) \leq \min\{S_i^V(A, B), S_i^V(B, C)\}.$$

$$\text{Hence } S_0^\omega(A, C) = \sum_{i=1}^n \omega_i S_i^H(A, C) \times S_i^V(A, C)$$

$$\leq \sum_{i=1}^n \omega_i \min\{S_i^H(A, B) \times S_i^V(A, B), S_i^H(B, C) \times S_i^V(B, C)\}$$

$$\leq \sum_{i=1}^n \omega_i \min\{S_i^H(A, B) \times S_i^V(A, B), S_i^H(B, C) \times S_i^V(B, C)\} = \min\{S_0^\omega(A, B), S_0^\omega(B, C)\}. \square$$

Example 1. Suppose that A and B are two IFs in $X = \{x_1, x_2, x_3, x_4\}$ where

$$A = \{(x_1, 0.3, 0.4), (x_2, 0.9, 0.05), (x_3, 0.7, 0.2), (x_4, 0.6, 0.3)\}$$

$$B = \{(x_1, 0.3, 0.2), (x_2, 0.7, 0.1), (x_3, 0.3, 0.6), (x_4, 0.5, 0.45)\}.$$

We have:

$$+ S_0(A, B) = 0.687275 ;$$

$$+ \text{ With the weight vector } \omega = (0.35, 0.4, 0.1, 0.15) , \text{ then } S_0^\omega(A, B) = 0.746704.$$

Now, we will compare the new similarity measure to other ones. Given $X = \{x_1, x_2, \dots, x_n\}$ is a universal set, let $A = \{(x_i, \mu_A(x_i), \nu_A(x_i)) \mid x_i \in X\}$, $B = \{(x_i, \mu_B(x_i), \nu_B(x_i)) \mid x_i \in X\}$ be two IF setson X .

We recall some existing similarity measures as follows:

+ The similarity measure of Ye (2011)

$$C_1(A, B) = \frac{1}{n} \sum_{i=1}^n \frac{\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i)}{\sqrt{\mu_A^2(x_i) + \nu_A^2(x_i)}\sqrt{\mu_B^2(x_i) + \nu_B^2(x_i)}}.$$

+ The similarity measure of Shi and Ye (2013)

$$C_2(A, B) = \frac{1}{n} \sum_{i=1}^n \frac{\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i) + \pi_A(x_i)\pi_B(x_i)}{\sqrt{\mu_A^2(x_i) + \nu_A^2(x_i) + \pi_A^2(x_i)}\sqrt{\mu_B^2(x_i) + \nu_B^2(x_i) + \pi_B^2(x_i)}}.$$

+ The similarity measure of Ye (2016)

$$CS_1(A, B) = \frac{1}{n} \sum_{i=1}^n \cos \left[\frac{\pi}{2} \max \left(\left| \mu_A(x_i) - \mu_B(x_i) \right|, \left| \nu_A(x_i) - \nu_B(x_i) \right|, \left| \pi_A(x_i) - \pi_B(x_i) \right| \right) \right];$$

$$CS_2(A, B) = \frac{1}{n} \sum_{i=1}^n \cos \left[\frac{\pi}{4} \left(\left| \mu_A(x_i) - \mu_B(x_i) \right| + \left| \nu_A(x_i) - \nu_B(x_i) \right| + \left| \pi_A(x_i) - \pi_B(x_i) \right| \right) \right].$$

However, one can find some drawbacks of the cosine similarity measure as follows:

Example 2. On the universal set $X = \{x_1, x_2\}$, we consider $A = \{(x_1, 0.5, 0), (x_2, 0, 0)\}$, $B = \{(x_1, 1, 0), (x_2, 1, 0)\}$ and $C = \{(x_1, 0.5, 0.5), (x_2, 0, 1)\}$.

It is obviously that $C_1(A, B), C_1(A, C)$ are not determine. Moreover, we easy see that $C \subset A \subset B$. This implies $C_2(B, C) \leq \min\{C_2(A, B), C_2(A, C)\}$, but we have $C_2(B, C) = \frac{\sqrt{0.5}}{2}$, $C_2(A, B) = \frac{\sqrt{0.5}}{2}$ and $C_2(A, C) = \frac{0.5}{2} = 0.25$. It means

$$C_2(B, C) = \frac{\sqrt{0.5}}{2} > \min\{C_2(A, B), C_2(A, C)\} = 0.25.$$

Thus the measure of Shi and Ye (2013) does not satisfy the condition (iv) in Definition 4 (Section 2).

Meanwhile, using the proposed new measure in Eq. (1) we have $S_0(B, C) = 0.1516$, $S_0(A, C) = 0.25$ and $S_0(A, B) = 0.4872$ which satisfies $C_2(B, C) \leq \min\{C_2(A, B), C_2(A, C)\}$.

In the pattern recognition, there are two patterns A_1, A_2 and a sample B . If $S(A_1, B) < S(A_2, B)$ then we put B belongs to pattern A_2 . We consider the following example:

Example 3. On the universal set $X = \{x_1, x_2\}$, we consider two patterns A_1, A_2 as follows

$$A_1 = \{(x_1, 0.5, 0), (x_2, 0, 0)\}, A_2 = \{(x_1, 0.5, 0.5), (x_2, 0, 1)\}$$

$$B = \{(x_1, 1, 0), (x_2, 1, 0)\}.$$

Question: which sample B belongs to?

If we follow the similarity measure of Ye (2016) then

$$CS_1(B, A_1) = CS_1(B, A_2) = \frac{\sqrt{0.5}}{2}, CS_2(B, A_1) = CS_2(B, A_2) = \frac{\sqrt{0.5}}{2}.$$

These do not show B belongs to pattern A_1 or A_2 . On the other hand, if using our method in Equation (2), then we have $S_0(B, A_2) = 0.1516$, $S_0(B, A_1) = 0.4872$ and thus B belongs to pattern A_1 . This is justified because $A_2 \subset A_1 \subset B$.

The above examples show the advantages of the new measure in this paper compared to some previous measures.

3. The similarity measure of the IVIFSs

We will extend our method to the IVIFSs in this section. Let A and B be two arbitrary IVIFSs in X as follows:

$$A = \left\{ \left(x_i, \left[\mu_A^L(x_i), \mu_A^U(x_i) \right], \left[v_A^L(x_i), v_A^U(x_i) \right] \right) \mid x_i \in X \right\};$$

$$B = \left\{ \left(x_i, \left[\mu_B^L(x_i), \mu_B^U(x_i) \right], \left[v_B^L(x_i), v_B^U(x_i) \right] \right) \mid x_i \in X \right\}.$$

For $s, t \in \mathbb{N} = \{0, 1, 2, \dots\}$, we denote

$$S_i^{\mu}(A, B) = e^{-\frac{|\mu_A^L(x_i) - \mu_B^L(x_i)|^s + |\mu_A^U(x_i) - \mu_B^U(x_i)|^s}{2}}$$

$$S_i^{\nu}(A, B) = 1 - \frac{|\nu_A^L(x_i) - \nu_B^L(x_i)|^t + |\nu_A^U(x_i) - \nu_B^U(x_i)|^t}{2} \text{ for all } i = 1, 2, \dots, n.$$

Definition 7. We define the similarity measure of two intervals IF A and B as follows

$$S_{s,t}^{\omega}(A, B) = \frac{1}{n} \sum_{i=1}^n S_i^{\mu}(A, B) \times S_i^{\nu}(A, B). \tag{3}$$

Theorem 3. Let A and B be two arbitrary IVIFSs on X . We have

- (i) $0 \leq S_{s,t}^{\omega}(A, B) \leq 1, \forall A, B \in IVIFS(X)$;
- (ii) $S_{s,t}^{\omega}(A, B) = S_{s,t}^{\omega}(B, A), \forall A, B \in IVIFS(X)$;
- (iii) $S_{s,t}^{\omega}(A, A) = 1, \forall A \in IVIFS(X)$;
- (iv) For all $A, B, C \in IVIFS(X)$ such that $A \subseteq B \subseteq C$ we have $S_{s,t}^{\omega}(A, C) \leq S_{s,t}^{\omega}(A, B)$ and $S_{s,t}^{\omega}(A, C) \leq S_{s,t}^{\omega}(B, C)$.

Proof. It is easy. \square

In the case, each element x_i in the universal $X = \{x_1, x_2, \dots, x_n\}$ is assigned with a weight $\omega_i \in [0, 1] (i = 1, 2, \dots, n)$ and $\sum_{i=1}^n \omega_i = 1$ then the similarity measure of A and B is defined by:

Definition 8. We define the similarity measure of two intervals IF A and B as follows

$$S_{s,t}^{\omega}(A, B) = \sum_{i=1}^n \omega_i S_i^{\mu}(A, B) \times S_i^{\nu}(A, B). \tag{4}$$

Remark. It is easy to show that the SM of A and B in the Definition 8 satisfies all the conditions of the Theorem 3.

Example 4. We consider a pattern recognition problem about the classification of minerals; the data was quoted from Liu et al. (2016). There are three classes of given minerals, which are expressed by the IVIFSs A_1, A_2, A_3 in the feature space $X = \{x_1, x_2, x_3, x_4, x_5, x_6\}$. Assume there is a new mineral A along with its known attribute values. Our aim is to determine which class that A belongs to. The descriptive data information is given in Table 1. According to the Table 2, we can comment that the similarity measure between $A_i (i = 1, 2, 3)$ and A with $\omega = (\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6})$ is $S(A_3, A) > S(A_1, A) > S(A_2, A)$, so A belongs to class A_3 .

Table 1. Data information of minerals

	A_1	A_2	A_3	A
x_1	([0.72,0.74], [0.1,0.12])	([0.42,0.45], [0.38,0.40])	([0.30,0.32], [0.45,0.47])	([0.60,0.63], [0.30,0.35])
x_2	([0.00,0.05], [0.80,0.82])	([0.65,0.67], [0.28,0.30])	([0.90,1.00], [0.00,0.00])	([0.50,0.53], [0.34,0.36])
x_3	([0.18,0.20], [0.62,0.63])	([0.00,1.00], [0.00,0.00])	([0.18,0.20], [0.70,0.73])	([0.20,0.21], [0.68,0.70])
x_4	([0.49,0.50], [0.35,0.37])	([0.70,0.90], [0.00,0.10])	([0.15,0.16], [0.75,0.78])	([0.20,0.22], [0.75,0.77])
x_5	([0.01,0.02], [0.60,0.63])	([0.80,1.00], [0.00,0.00])	([0.00,0.05], [0.88,0.90])	([0.05,0.07], [0.87,0.90])
x_6	([0.72,0.74], [0.12,0.13])	([0.90,1.00], [0.00,0.00])	([0.65,0.68], [0.25,0.30])	([0.65,0.70], [0.25,0.30])

Table 2. The similarity measure between A_i ($i = 1, 2, 3$) and A

	$S(A_1, A)$	$S(A_2, A)$	$S(A_3, A)$
The similarity using Eq.(3)	0.583615	0.353333	0.744232

Table 3. Correlation coefficients between A_i ($i = 1, 2, 3$) and A under different methods

	$\rho(A_1, A)$	$\rho(A_2, A)$	$\rho(A_3, A)$
Method in Li & Cheng, 2002	0.53	-0.52	0.81
Method in Bustince & Burillo, 1995	0.86	0.52	0.94
Method in Shidpour, Bernard & Shahrokhi, 2013	0.78	0.77	0.84
Method in Hung, 2001	0.52	-0.53	0.81
Method in Buhaesku, 1998	0.85	0.51	0.94

We also compare the result of our method to the results obtained by other methods in Li and Cheng (2002), Bustince and Burillo (1995), Shidpour, Bernard, and Shahrokhi (2013), Hung (2001) and Buhaesku (1988) (see Table 3). Our result is identical with the results using these methods.

4. Multi-criteria decision making model

In the MCDM problem, the decision makers are responsible for choosing the best suitable option from the set of alternatives. They must assess alternatives under criteria. The criterion outcomes are a basis to rank the options (Kelemenis & Askounis, 2010). The fundamental process of MCDM model includes the following steps (Kelemenis & Askounis, 2010; Shen, Olfat, Govindan, Khodaverdi & Diabat, 2013):

- Step 1. Construct the decision matrix.
- Step 2. Determine the matrix of the criteria weight.
- Step 3. Construct the weighted decision matrix.
- Step 4. Rank the alternatives.

The object of this section is applying the similarity measure for the interval-valued intuitionistic fuzzy sets to construct a novel MCDM model.

Let $\{P_i \ (i = 1, 2, \dots, m)\}$ be a set of alternatives for evaluation, $C_j \ (j = 1, 2, \dots, n)$ be n criteria to select the optimal option. There are h decision makers (DMs) $D_t \ (t = 1, 2, \dots, h)$ to identify

the importance of criteria and the meeting criteria of alternatives through the linguistic terms given in the Table 4 and Table 5. The SM of the interval-valued IF sets is used to rank the alternatives. The model has the following steps:

Table 4. The intuitionistic rating scale for important degree

Linguistic scale for important degree	Interval intuitionistic fuzzy scale
Unimportant (UI)	([0.2, 0.3], [0.5, 0.6])
Ordinary Important (OI)	([0.3, 0.4], [0.4, 0.5])
Important (I)	([0.5, 0.6], [0.3, 0.4])
Very Important (VI)	([0.7, 0.8], [0.1, 0.2])
Absolutely Important (AI)	([0.8, 0.9], [0, 0.1])

Table 5. The intuitionistic rating scale

Linguistic scale for perform an alternatives	Interval intuitionistic fuzzy scale
Very low (VL)	([0.1, 0.2], [0.6, 0.7])
Low (L)	([0.3, 0.4], [0.5, 0.6])
Fair (F)	([0.4, 0.5], [0.4, 0.5])
Good (G)	([0.6, 0.7], [0.2, 0.3])
Very good (VG)	([0.7, 0.8], [0.1, 0.2])

Step 1. Determine the weight vector ω_j of the criterion $C_j (j = 1, 2, \dots, n)$.

Each of criterion has a different importance, which is made by the DMs. Assume that the aggregated weight of the criterion C_j is represented by $\omega_j = ([a_j, b_j], [c_j, d_j])$.

Step 2. Evaluate the performance of the alternative P_i under the criterion C_j .

The DMs appraise alternatives with respect to all criteria. The aggregated value of each alternative is expressed by the interval-valued intuitionistic fuzzy number $P_{ij} = ([\mu^L_{ij}, \mu^U_{ij}], [v^L_{ij}, v^U_{ij}])$, $i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$.

The criteria are divided into two types, benefit or cost, therefore the cost criteria need to be transformed into benefit type. The P_{ij} is converted to the standardized value r_{ij} by $r_{ij} = P_{ij}$ if C_j is benefit type, $r_{ij} = P^c_{ij}$ if C_j is cost type (According to Xu & Hu, 2010).

Step 3. Determine the overall values of alternatives

The overall value of each alternative with respect to each criterion is multiple of the value r_{ij} with the aggregated weight ω_j

$$V_{ij} = r_{ij} \otimes \omega_j, i = 1, 2, \dots, m; j = 1, 2, \dots, n. \text{ (According to Shen et al., 2013).} \tag{5}$$

Step 4. Rank the alternatives

In order to identify the optimal alternative, all alternatives need to be compared with the ideal solution $P_b = ([1, 1], [0, 0])$. The alternatives are ranked via the similarity measure

$$S_i = \frac{1}{n} \sum_{j=1}^n S_i^H(V_{ij}, P_b) \times S_i^Y(V_{ij}, P_b), (i = 1, 2, \dots, m). \tag{6}$$

Following the property of the similarity measure, the alternative A_i is better than the alternative A_k if $S_i > S_k$, $i = 1, 2, \dots, m$ and $k = 1, 2, \dots, m$.

5. Case study

In this section, a case study was performed in a well-known company in Vietnam, which is a joint venture company between Japan, Thailand and Vietnam covertures. This company has grown rapidly and become one of the leading companies in the automobile and motorcycles sector. The company put an acute focus into providing customers with high quality products at reasonable prices. The managers of this company involve how to select the target market to gain outstanding profit. In order to determine the most suitable market, we conducted a survey on target market research. The four market segments defined as alternatives were denoted by SE_1 , SE_2 , SE_3 , and SE_4 . The data used as input for evaluated process was collected by interview the experts. This group included the marketing manager, the sales manager, the experts in business and the expert in university.

Simkin and Dibb (1998) carried out a survey of the criteria for selecting target markets. They referred from literature and interviewed with experts. As a result, Identify profitability (C1), The growth of the market (C2), Size of market (C3), likely customer satisfaction (C4), sales volume (C5), Likelihood of sustainable differential advantage (C6), Development opportunities (C7) and The differentiation of product (C8) were most heavily-used criteria. The experts recognized that these criteria were suitable in Vietnam’s market. In this section, we will use these criteria to select the target market. The procedure is represented as follow:

Step 1. Determining the weight of criterion.

After defining the criteria, the experts were asked to determine the important degree of each criterion using the scale for importance. Based on the opinion of experts for each criterion, we calculated their averaged values. The weight of each criterion is shown in the Table 6.

Step 2. Evaluating the performance of the market segment with respect to the criterion.

Four experts are responsible for evaluating the degree of satisfying of alternatives respect to the criteria. Their opinions were expressed through the interval intuitionistic fuzzy numbers, which is shown in the Table 5. The averaged values are assigned by the decision makers for markets under the criteria is shown in the Table 7.

Table 6. The weight of criterion

Criteria	Decision makers ($D_1 - D_4$)				Averaged values
C_1	AI	AI	AI	AI	([0.8, 0.9], [0, 0.1])
C_2	VI	VI	VI	I	([0.659, 0.762], [0.132, 0.238])
C_3	VI	I	VI	VI	([0.659, 0.762], [0.132, 0.238])
C_4	OI	I	I	I	([0.456, 0.557], [0.322, 0.423])
C_5	I	I	OI	OI	([0.408, 0.510], [0.346, 0.447])
C_6	I	I	I	I	([0.5, 0.6], [0.3, 0.4])
C_7	I	I	VI	VI	([0.613, 0.717], [0.173, 0.283])
C_8	I	I	I	I	([0.5, 0.6], [0.3, 0.4])

In this case, all the criteria used for evaluation are the criteria of benefit. Therefore, it is not necessary to convert to the standardized values.

Step 3. Determining the overall values of market segments.

The overall value of each market segment with respect to each criterion is calculated by the Equation (5), it is shown in the last column of the Table 7.

Step 4. Calculating the similarity measure S_j of each market by Eq. (6) with $s = t = 1$.

Ranking of markets based on the similarity measure. The result is shown in the Table 8 and Figure 1.

Table 7. The interval-valued intuitionistic fuzzy decision matrix

Seg- ment	Crite- ria	Decision makers				Averaged values	Overall values V_{ij}
		VG	VG	G	G		
SE_1	C_1	VG	VG	G	G	([0,654,0.755], [0.141,0.245])	([0.523, 0.68], [0.141, 0.321])
	C_2	G	G	VG	VG	([0,654,0.755], [0.141, 0.245])	([0.431, 0.576], [0.254, 0.425])
	C_3	G	VG	G	F	([0.588,0.692], [0,200, 0.308])	([0.388, 0.527], [0.305, 0.473])
	C_4	F	G	F	F	([0.476,0,56], [0.336, 0.440])	([0.209, 0.312], [0.550, 0.677])
	C_5	G	F	F	F	([0.476,0,56], [0.336, 0.440])	([0.187, 0.286], [0.566, 0.691])
	C_6	VG	G	VG	VG	([0.678,0.779], [0.119, 0.221])	([0.339, 0.467], [0.383, 0.533])
	C_7	G	G	VG	G	([0,628,0.729], [0.168, 0.271])	([0.385, 0.523], [0.312, 0.477])
	C_8	G	G	VG	VG	([0.654,0.755], [0.141, 0.245])	([0.327, 0.453], [0.399, 0.547])
SE_2	C_1	G	F	G	G	([0,557,0.659], [0.238, 0.341])	([0.446, 0.593], [0.238, 0.407])
	C_2	G	F	F	F	([0,458,0.56], [0.336, 0.440])	([0.302, 0.427], [0.424, 0.573])
	C_3	F	G	G	F	([0.510,0.613], [0.283, 0.387])	([0.336, 0.467], [0.377, 0.533])
	C_4	F	F	F	F	([0.4, 0.5], [0.4, 0.5])	([0.182, 0.279], [0.593, 0.712])
	C_5	G	F	F	F	([0.458,0.56], [0.336,0.440])	([0.187, 0.286], [0.566, 0.691])
	C_6	G	G	G	F	([0.557, 0.659], [0,238, 0.341])	([0.279, 0.396], [0.467, 0.605])
	C_7	G	G	F	F	([0.510,0.613], [0.283, 0.387])	(0.313, 0.439], [0.407, 0.561])
	C_8	G	F	F	G	([0.510,0.613], [0.283, 0.387])	([0.255, 0.368], [0.498, 0.632])
SE_3	C_1	G	G	G	VG	([0.628, 0.729], [0.168,0.271])	([0.502, 0.656], [0.168, 0.344])
	C_2	VG	VG	VG	VG	([0.7, 0.8], [0.1, 0.2])	([0.461, 0.61], [0.218, 0.390])
	C_3	VG	VG	G	G	([0.654,0.755], [0.141, 0.245])	([0.431, 0.576], [0.254, 0.425])
	C_4	G	VG	VG	VG	([0.678,0.779], [0.119,0.221])	([0.309, 0.434], [0.403, 0.551])
	C_5	VG	VG	VG	VG	([0.7, 0.8], [0.1, 0.2])	([0.286, 0.408], [0.412, 0.558])
	C_6	G	G	VG	G	([0.673, 0.729], [0.228, 0.336])	([0.314, 0.437], [0.418, 0.563])
	C_7	G	G	VG	VG	([0.654,0.755], [0.141, 0.245])	([0.401, 0.542], [0.290, 0.459])
	C_8	G	VG	VG	G	([0.654,0.755], [0.141, 0.245])	([0.327, 0.453], [0.399, 0.547])
SE_4	C_1	VG	VG	G	G	([0.654,0.755], [0.141, 0.245])	([0.523, 0.68], [0.141, 0.321])
	C_2	VG	VG	G	G	([0.654,0.755], [0.141, 0.245])	([0.431, 0.576], [0.254, 0.425])
	C_3	G	G	G	VG	([0,628,0.729], [0.228, 0.336])	([0.414, 0.556], [0.278, 0.444])
	C_4	F	F	F	F	([0.4, 0.5], [0.4, 0.5])	([0.182, 0.279], [0.593, 0.712])
	C_5	G	F	F	F	([0.476,0,56], [0.336, 0.440])	([0.187, 0.286], [0.566, 0.691])
	C_6	VG	VG	G	G	([0.654,0.755], [0.141, 0.245])	([0.327, 0.453], [0.399, 0.547])
	C_7	F	F	G	G	([0.510,0.614], [0.283, 0.387])	([0.313, 0.439], [0.407, 0.561])
	C_8	F	L	L	L	([0.327, 0.427], [0.473,0.573])	([0.163, 0.256], [0.631, 0.744])

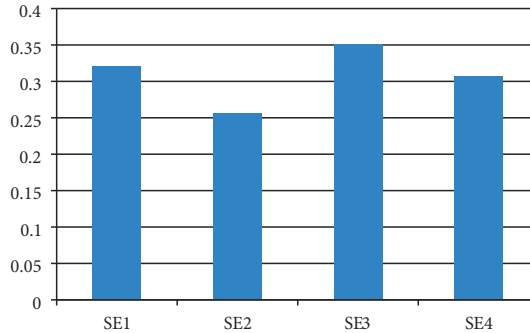


Figure 1. The similarity measure of four market segments with the best alternative

Table 8. The result of ranking markets based on the similarity measure

The market	SE1	SE2	SE3	SE4
Similarity measure	0.32075	0.25659	0.35034	0.30735
Ranking	2	4	1	3

The ranking between markets is SE₃, SE₁, SE₄, SE₂, the target market is SE₃.

6. Discussion

In order to check our methods as well as measure, we consider the models of Shen et al. (2013); Xu (2007b); Wei, Wang, and Zhang (2011); the similarity measures of Ye (2011, 2016); Shi and Ye (2013).

In the model of Shen et al. (2013), the TOPSIS method was used to rank the options, the evaluated values were expressed by the triangular fuzzy numbers and the distance function

$$d(A, B) = \sqrt{\frac{1}{3} \left[(a_1 - a_2)^2 + (b_1 - b_2)^2 + (c_1 - c_2)^2 \right]}$$

In the model of Xu (2007b), the similarity measure was used in the TOPSIS model to rank the alternatives with

$$s(A, B) = 1 - \left[\frac{1}{2n} \sum_{i=1}^n \left(|\mu_{iA} - \mu_{iB}|^2 + |v_{iA} - v_{iB}|^2 + |\pi_{iA} - \pi_{iB}|^2 \right) \right]^{1/2}, \pi_i = 1 - \mu_i - v_i.$$

The evaluated values used the intuitionistic fuzzy numbers.

In the model of Wei et al. (2011), the similarity measure was used in the TOPSIS model to rank the alternatives with

$$s(A, B) = \frac{1}{n} \sum_{i=1}^n \frac{2 - \min\{u_i^-, v_i^-\} - \min\{u_i^+, v_i^+\}}{2 + \min\{u_i^-, v_i^-\} + \min\{u_i^+, v_i^+\}}$$

in which $u_i^- = |u_{iA}^L - u_{iB}^L|, u_i^+ = |u_{iA}^U - u_{iB}^U|, v_i^- = |v_{iA}^L - v_{iB}^L|, v_i^+ = |v_{iA}^U - v_{iB}^U|$. The interval-valued intuitionistic fuzzy numbers were used to represent the evaluated values.

The similarity measures of Ye (2011, 2016), Shi and Ye (2013) were used to arrange the order of alternatives. The results of ranking are shown in the Table 9.

Table 9. The results of ranking of different methods

Methods	Ranking order results
Xu (2007b)'s method	SE_3, SE_1, SE_4, SE_2
Shen et al. (2013)'s method	SE_3, SE_1, SE_4, SE_2
Ye (2011)'s measure	SE_3, SE_1, SE_4, SE_2
Shi and Ye (2013)'s measure	SE_3, SE_1, SE_4, SE_2
Ye (2016)'s measure	SE_3, SE_1, SE_4, SE_2
Proposed method	SE_3, SE_1, SE_4, SE_2
Wei et al (2011)'s method	SE_1, SE_3, SE_4, SE_2

The result of our method is as same as with the methods of Xu (2007b), Shen et al. (2013), Ye (2011, 2016) and Shi and Ye (2013) There is a little difference between the ranking in the method of Wei et al. (2011) and the rest of methods. On the other hand, based on the evaluation of the decision makers in the third column of the Table 7, the SE_3 is better than the SE_1 in the almost of criteria. Thus, the ranking SE_3, SE_1, SE_4, SE_2 is reliable.

Conclusions

In current markets, the businesses have to face a lot of challenges due to the development of new technologies, competitors and customers evolving trends. The target market is the center of the marketing decisions. It helps the firms provide suitable marketing strategies. In this article, we have introduced some formulas that define the similarity measures between intuitionistic fuzzy sets and interval-valued intuitionistic fuzzy sets. A multi-criteria decision making method is proposed under intuitionistic fuzzy environment to select the best alternatives. Finally, a case study for selecting the target market was performed. The similarity measures are new and convenient for applications. Our model can be used to rank alternatives in other fields.

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