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PROBABILISTIC AGGREGATION OPERATORS AND THEIR APPLICATION IN UNCERTAIN MULTI-PERSON DECISION-MAKING

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Abstract. We present the uncertain probabilistic ordered weighted averaging (UPOWA) operator. It is an aggregation operator that uses probabilities and OWA operators in the same formulation considering the degree of importance of each concept in the analysis. Moreover, it also uses uncertain information assessed with interval numbers in the aggregation process. The main advantage of this aggregation operator is that it is able to use the attitudinal character of the decision maker and the available probabilistic information in an environment where the information is very imprecise and can be assessed with interval numbers. We study some of its main properties and particular cases such as the uncertain probabilistic aggregation (UPA) and the uncertain OWA (UOWA) operator. We also develop an application of the new approach in a multi-person decision-making problem in political management regarding the selection of monetary policies. Thus, we obtain the multi-person UPOWA (MP-UPOWA) operator. We see that this model gives more complete information of the decision problem because it is able to deal with decision making problems under uncertainty and under risk in the same formulation.

Keywords: OWA operator, probabilities, probabilistic OWA operator, uncertainty, interval numbers, multi-person decision-making, policy management.

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1. Introduction

Decision making problems are very common in our lives because people are always making decisions. In the literature, we find a wide range of methods and theories for dealing with the decision process (Antuchevičienė *et al.* 2010; Brauers and Zavadskas 2010; Keršulienė *et al.* 2010; Liu 2009, 2011; Podvezko 2009; Xu 2010; Zavadskas *et al.* 2009, 2010a; Zavadskas and Turskis 2010; Zhang and Liu 2010) . The use of probabilities and the ordered weighted averaging (OWA) operator (Yager 1988) in the same aggregation process is a very useful method for considering the probabilistic information and the attitudinal character of the decision maker in the same formulation. Some studies have already considered this problem by referring to it as the immediate probability (Engemann *et al.* 1996; Merigó 2010; Yager *et al.* 1995). The main advantage of this approach is the possibility of underestimate or overestimate the probabilistic information according to the degree of orness (or optimism) given in the OWA operator. Thus, we are able to obtain a parameterized family of aggregation operators (Beliakov *et al.* 2007) between the maximum and the minimum. For further reading on the OWA operator, see for example (Chang and Wen 2010; Merigó and Casanovas 2010c; Merigó and Gil-Lafuente 2010, 2011; Wang *et al.* 2009; Yager 1998; Yager and Kacprzyk 1997; Zhou and Chen 2010). Note also that there exist in the literature other approaches that use probabilistic information and OWA operators in the same formulation including some decision making methods with Dempster-Shafer belief structure (Merigó and Casanovas 2009; Merigó *et al.* 2010; Yager 1992).

In this paper, it is worth noting the work by Xu and Da (2002) regarding the uncertain OWA (UOWA) operator. Basically, it is an aggregation operator that deals with uncertain information represented in the form of interval numbers. Since its introduction, several authors have developed further improvements. For example, Merigó and Casanovas (2011a) generalized it by using generalized and quasi-arithmetic means and developed several extensions with fuzzy and linguistic information (Merigó and Casanovas 2010a, 2010b). Wei (2009) developed a model with uncertain linguistic information and with intuitionistic fuzzy sets (Wei 2010a, 2010b; Wei *et al.* 2010).

The concept of immediate probability has some limitations. One of the most significant problems, as stated by Merigó (2009), is that it is not able to unify the probability and the OWA operator considering that sometimes one of them can be more relevant in the aggregation. Therefore, it is necessary to use another approach that it is able to unify both concepts but taking into account that they can be more or less relevant depending on the problem considered. For doing so, Merigó (2009) has suggested the probabilistic OWA (POWA) operator. It is a new aggregation operator that unifies the probability and the OWA operator giving different degrees of importance to each concept according to their relevance in the specific problem considered.

The POWA operator is very useful to unify the probability with the OWA operator when using exact numbers in the aggregation process. However, many situations of the real world cannot be assessed with exact numbers because the information is uncertain and very complex. Therefore, it is necessary to use another approach that it is able to assess this situation such as the use of interval numbers. The interval numbers (Moore 1966) are a very useful technique for representing the uncertainty by considering the best and worst possible results that could happen in the environment and the most possible ones.

The aim of this paper is to present the uncertain probabilistic OWA (UPOWA) operator. It is an aggregation operator that uses uncertain information in the aggregation process by using interval numbers in the POWA operator. Therefore, we are able to assess the POWA operator considering the best and worst results that could happen in the aggregation process and some of the most possible ones. The main advantage of the UPOWA operator is that it provides more complete information to the decision maker by using interval numbers that includes a wide range of results and by using probabilities and OWA operators in the same formulation considering the degree of importance of each concept in the aggregation. Thus, we are able to consider objective information (probabilistic) and the attitudinal character of the decision maker in the same formulation. We study some of its main properties and particular cases including the UOWA operator, the uncertain average (UA), the uncertain probabilistic aggregation (UPA), the uncertain probabilistic maximum and the uncertain probabilistic minimum. Note that by using interval numbers we can represent all the possible results that may occur in the uncertain environment. Thus we can guarantee that at least we are considering all the possible situations without losing information in the analysis. However, as we are in uncertainty, we do not know which scenario will occur.

The other objective of this paper is to analyze the applicability of this new approach and we see that it is very broad because all the previous studies that use the probability or the OWA operator can be revised and extended with this new approach. For example, we can apply it in statistics, economics, engineering, physics and medicine. We focus in a multi-person decision making problem. We find a more general aggregation process that considers the opinion of several persons or experts in the analysis. We call it the multi-person UPOWA (MP-UPOWA) operator. We see that it also includes a wide range of particular cases including the multi-person UPA (MP-UPA), the multi-person UA (MP-UA) and the multi-person UOWA (MP-UOWA) operator. We implement the new approach in a decision making problem regarding the selection of optimal monetary policies.

This paper is organized as follows. In Section 2, we briefly review some basic concepts regarding the interval numbers, the UOWA operator and the POWA operator. In Section 3, we present the UPOWA operator and Section 4 analyzes a wide range of particular cases. Section 5 introduces a multi-person decision making application and Section 6 an illustrative example. Section 7 summarizes the main conclusions found in the paper.

2. Preliminaries

In this Section, we briefly describe the interval numbers, the UOWA operator and the POWA operator.

2.1. Interval Numbers

The interval numbers (Moore 1966) are a very useful and simple technique for representing the uncertainty. They have been used in an astonishingly wide range of applications.

The interval numbers can be expressed in different forms. For example, if we assume a 4-tuple (a_1, a_2, a_3, a_4), that is, a quadruplet; we could consider that a_1 and a_4 represents the minimum and the maximum of the interval number, and a_2 and a_3, the interval with the highest probability or possibility, depending on the use we want to give to the interval numbers. Note that $a_1 \leq a_2 \leq a_3 \leq a_4$. If $a_1 = a_2 = a_3 = a_4$, then, the interval number is an exact number; if $a_2 = a_3$, it is a 3-tuple known as triplet; and if $a_1 = a_2$ and $a_3 = a_4$, it is a simple 2-tuple interval number.

In the following, we are going to review some basic interval number operations as follows. Let A and B be two triplets, where $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$. Then:

$A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$;

$A - B = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$;

$A \times k = (k \times a_1, k \times a_2, k \times a_3)$; for $k > 0$;

$A \times B = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3)$; for R^+;

$A \times B = (\mathrm{Min}\{a_1 \times b_1, a_3 \times b_1, a_1 \times b_3, a_3 \times b_3\}, \mathrm{Max}\{a_1 \times b_1, a_3 \times b_1, a_1 \times b_3, a_3 \times b_3\})$; for R;

$A \div B = (a_1 \div b_3, a_2 \div b_2, a_3 \div b_1)$; for R^+;

$A \div B = (\mathrm{Min}\{a_1 \div b_1, a_3 \div b_1, a_1 \div b_3, a_3 \div b_3\}, \mathrm{Max}\{a_1 \div b_1, a_3 \div b_1, a_1 \div b_3, a_3 \div b_3\})$; for R.

Note that other operations could be studied (Moore 1966) but in this paper we will focus on these ones.

2.2. The Uncertain OWA Operator

The uncertain OWA (UOWA) operator was introduced by Xu and Da (2002). It is an extension of the OWA operator (Yager 1988) for uncertain situations where the available information can be assessed with interval numbers. It can be defined as follows:

Definition 1. Let Ω be the set of interval numbers. An UOWA operator of dimension n is a mapping $UOWA$: $\Omega^n \rightarrow \Omega$ that has an associated weighting vector W of dimension n such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, then:

$$UOWA\ (\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \sum_{j=1}^{n} w_j b_j, \tag{1}$$

where b_j is the jth largest of the $\tilde{a}_i$ and $\tilde{a}_i$ is the argument variable represented in the form of interval numbers.

Note also that different families of UOWA operators can be studied by choosing a different weighting vector such as the step-UOWA operator, the window-UOWA, the median-UOWA, the olympic-UOWA, the centered-UOWA and the S-UOWA.

2.3. The Probabilistic OWA Operator

The probabilistic ordered weighted averaging (POWA) operator is an aggregation operator that unifies the probability and the OWA operator in the same formulation considering the degree of importance that each concept has in the analysis (Merigó 2009). It is defined as follows.

Definition 2. A POWA operator of dimension n is a mapping $POWA$: $R^n \rightarrow R$ that has an associated weighting vector W of dimension n such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, according to the following formula:

$$POWA\ (a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} \hat{v}_j b_j, \tag{2}$$

where b_j is the jth largest of the a_i, each argument a_i has an associated weight (WA) v_i with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0, 1]$, $\hat{v}_j = \beta w_j + (1 - \beta) v_j$ with $\beta \in [0, 1]$ and v_j is the weight (WA) v_i ordered according to b_j, that is, according to the jth largest of the a_i.

By choosing a different manifestation in the weighting vector, we are able to obtain a wide range of particular types of POWA operators (Merigó 2009). Especially, when $\beta = 0$, we get the probabilistic aggregation, and if $\beta = 1$, we get the OWA operator.

3. The Uncertain Probabilistic OWA Operator

The uncertain probabilistic ordered weighted averaging (UPOWA) operator is an extension of the OWA operator for situations where we find probabilistic and uncertain information that can be assessed with interval numbers. Its main advantage is that it can unify both concepts considering the degree of importance that they have in the specific problem considered. Thus, we are able to apply this formulation to all the previous models that use probabilities or OWAs obtaining a more complete approach that it is able to consider a wide range of scenarios that includes the classical approaches. Specially, it is worth noting that in decision making problems, this approach is able to include decision making under risk and under uncertainty environments in the same formulation. This approach seems to be complete, at least as an initial real unification between OWA operators and probabilities.

However, it is worth noting that some previous models already considered the possibility of using OWA operators and probabilities in the same formulation. The main model is the concept of immediate probability (Engemann *et al.* 1996; Merigó 2010; Yager *et al.* 1995; Yager 1999). Although it seems to be a good approach it is not so complete than the UPOWA because it can unify OWAs and probabilities in the same model but it can not take in consideration the degree of importance of each case in the aggregation process. Before studying the UPOWA, we are going briefly to consider the immediate probabilities with interval numbers (IP-UOWA). For uncertain situations assessed with interval numbers, the immediate probability can be defined as follows.

Definition 3. Let Ω be the set of interval numbers. An IPUOWA operator of dimension n is a mapping $IPUOWA$: $\Omega^n \to \Omega$ that has associated a weighting vector W of dimension n such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, according to the following formula:

$$IPUOWA\ (\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \sum_{j=1}^{n} \hat{p}_j b_j, \tag{3}$$

where b_j is the jth largest of the $\tilde{a}_i$, the $\tilde{a}_i$ are interval numbers and each one has associated a probability p_i with $\sum_{i=1}^{n} p_i = 1$ and $p_i \in [0, 1]$, $\hat{p}_j = (w_j p_j / \sum_{j=1}^{n} w_j p_j)$ and p_j is the probability p_i ordered according to b_j, that is, according to the jth largest of the $\tilde{a}_i$.

Note that the IPUOWA operator is a good approach for unifying probabilities and OWAs in some particular situations. But it is not always useful, especially in situations where we want to give more importance to the OWA operators or to the probabilities. One way to see why this unification does not seem to be a final model is considering other ways of representing $\hat{p}_j$. For example, we could also use $\hat{p}_j = [w_j + p_j / \sum_{j=1}^{n} (w_j + p_j)]$ or other similar approaches.

Note that other approaches that could be taken into account are the hybrid averaging (HA) (Xu and Da 2003; Zhao *et al.* 2009, 2010) and the weighed OWA (WOWA) operator (Torra 1997; Torra and Narukawa 2007). These models unify the OWA operator with the weighted average (WA). Therefore, they can also be extended for situations with the OWA operator and probabilities assuming that for some situations the WA can be seen as a probability, for example, when we use the WA as a subjective probability. As said before, these an other approaches are useful for some particular situations but they does not seem to be so complete than the UPOWA because they can unify OWAs with probabilities (or with WAs) but they can not unify them giving different degrees of importance to each case. Note that in future research we will also prove that these models can be seen as a special case of a general UPOWA operator (or its respective model with WAs) that uses quasi-arithmetic means. Obviously, it is possible to develop more complex models of the IP-UOWA, the HA (or uncertain HA) and the WOWA that takes into account the degree of importance of the OWAs and the probabilities (or WAs) in the model but they seem to be artificial and not a natural unification as it will be shown below. In the following, we are going to analyze the UPOWA operator. It can be defined as follows.

Definition 4. Let Ω be the set of interval numbers. An UPOWA operator of dimension n is a mapping $UPOWA: \Omega^n \rightarrow \Omega$ that has associated a weighting vector W of dimension n such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, according to the following formula:

$$UPOWA\ (\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \sum_{j=1}^{n} \hat{p}_j b_j , \tag{4}$$

where b_j is the jth largest of the $\tilde{a}_i$, the $\tilde{a}_i$ are interval numbers and each one has an associated probability p_i with $\sum_{i=1}^{n} p_i = 1$ and $p_i \in [0, 1]$, $\hat{p}_j = \beta w_j + (1-\beta) p_j$ with $\beta \in [0, 1]$ and p_j is the probability p_i ordered according to b_j, that is, according to the jth largest of the $\tilde{a}_i$.

Note that it is also possible to formulate the UPOWA operator separating the part that strictly affects the OWA operator and the part that affects the probabilities. This representation is useful to see both models in the same formulation but it does not seem to be as a unique equation that unifies both models.

Definition 5. Let Ω be the set of interval numbers. An UPOWA operator is a mapping $UPOWA: \Omega^n \rightarrow \Omega$ of dimension n, if it has associated a weighting vector W, with $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0, 1]$ and a probabilistic vector V, with $\sum_{i=1}^{n} p_i = 1$ and $p_i \in [0, 1]$, such that:

$$UPOWA\ (\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \beta \sum_{j=1}^{n} w_j b_j + (1-\beta) \sum_{i=1}^{n} p_i \tilde{a}_i , \tag{5}$$

where b_j is the jth largest of the arguments $\tilde{a}_i$, the $\tilde{a}_i$ are interval numbers and $\beta \in [0, 1]$.

Note that if the weights of the probabilities and the OWAs are also uncertain, then, we have to establish a method for dealing with these uncertain weights. Note that in these situations it is very common that $W = \sum_{j=1}^{n} \tilde{w}_j \neq 1$ and $P = \sum_{i=1}^{n} \tilde{p}_i \neq 1$. Thus, a very useful method for dealing with these situations is by using:

$$UPOWA\ (\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \frac{\beta}{W} \sum_{j=1}^{n} \tilde{w}_j b_j + \frac{(1-\beta)}{P} \sum_{i=1}^{n} \tilde{p}_i \tilde{a}_i . \tag{6}$$

In the following, we are going to give a simple example of how to aggregate with the UPOWA operator. We consider the aggregation with both definitions. For simplicity, we assume that the weights are exact values.

Example 1. Assume the following arguments in an aggregation process: ([20, 30], [40, 50], [50, 60], [30, 40]). Assume the following weighting vector W = (0.2, 0.2, 0.2, 0.4) and the following probabilistic weighting vector P = (0.4, 0.3, 0.2, 0.1). Note that the probabilistic information has a degree of importance of 60% while the weighting vector W a degree of 40%. If we want to aggregate this information by using the UPOWA operator, we will get the following. The aggregation can be solved either with the Eq. (4) or (5). With Eq. (4) we calculate the new weighting vector as:

$$\hat{v}_1 = 0.4\times0.2+0.6\times0.2 = 0.2\,,\ \hat{v}_2 = 0.4\times0.2+0.6\times0.3 = 0.26,$$

$$\hat{v}_3 = 0.4\times0.2+0.6\times0.1 = 0.14\,,\ \hat{v}_4 = 0.4\times0.4+0.6\times0.4 = 0.4.$$

And then, we calculate the aggregation process as follows:

UPOWA = 0.2 ×[50, 60] + 0.26 ×[40, 50] + 0.14 ×[30, 40] + 0.4 ×[20, 30] = [32.6, 42.6].

With Eq. (5), we aggregate as follows:

UPOWA = 0.4 ×(0.2 ×[50, 60] + 0.2 ×[40, 50] + 0.2 ×[30, 40] + 0.4 ×[20, 30]) + 0.6 ×(0.4 ×[20, 30] + 0.3 ×[40, 50] + 0.2 ×[50, 60] + 0.1 ×[30, 40]) = [32.6, 42.6].

Obviously, we get the same results with both methods.

Note that different types of interval numbers could be used in the aggregation such as 2-tuples, triplets, quadruplets, etc. When using interval numbers in the UPOWA operator, we have the additional problem of how to reorder the arguments because now we are using interval numbers. Thus, in some cases, it is not clear which interval number is higher, so we need to establish an additional criteria for reordering the interval numbers. For simplicity, we recommend the following criteria. First, we analyze if there is an order between the interval numbers. That is, if all the values of the interval $A = (a_1, a_2, a_3)$ are higher than the values in the interval $C = (c_1, c_2, c_3)$ such that $a_1 > c_3$. If not, we will calculate an average of the interval number. For example, if $n = 2$, $(a_1 + a_2) / 2$; if $n = 3$, $(a_1 + 2a_2 + a_3) / 4$; and so on. In the case of tie, we will select the interval with the lowest increment $(a_2 - a_1)$. For 3-tuples and more we will select the interval with the highest central value.

From a generalized perspective of the reordering step, it is possible to distinguish between the descending UPOWA (DUPOWA) and the ascending UPOWA (AUPOWA) operator by using $w_j = w^*_{n-j+1}$, where w_j is the jth weight of the DUPOWA and w^*_{n-j+1} the jth weight of the AUPOWA operator.

If B is a vector corresponding to the ordered arguments b_j, we shall call this the ordered argument vector and W^T is the transpose of the weighting vector, then, the UPOWA operator can be expressed as:

$$UPOWA\ (\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = W^T B. \qquad (7)$$

A further interesting result consists in using infinitary aggregation operators (Mesiar and Pap 2008). Thus, we can represent an aggregation process where there are an unlimited number of arguments that appear in the aggregation process. Note that $\sum_{j=1}^{\infty} \hat{p}_j = 1$. By using, the UPOWA operator we get the infinitary UPOWA (∞-UPOWA) operator as follows:

$$\infty\text{–}UPOWA\ (\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \sum_{j=1}^{\infty} \hat{p}_j b_j. \qquad (8)$$

The reordering process is very complex because we have an unlimited number of arguments, so we never know which argument is the first one to be aggregated. For further reading on the usual OWA, see Mesiar and Pap (2008).

The UPOWA is monotonic, commutative, bounded and idempotent. It is monotonic because if $\tilde{a}_i \geq \tilde{u}_i$, for all $\tilde{a}_i$, then, $UPOWA\ (\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \geq UPOWA\ (\tilde{u}_1, \tilde{u}_2, \ldots, \tilde{u}_n)$. It is commutative because any permutation of the arguments has the same evaluation. That is, $UPOWA\ (\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = UPOWA\ (\tilde{u}_1, \tilde{u}_2, \ldots, \tilde{u}_n)$, where $(\tilde{u}_1, \tilde{u}_2, \ldots, \tilde{u}_n)$ is any permutation of the arguments ($\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n$). It is bounded because the UPOWA aggregation is delimitated by the minimum and the maximum: Min$\{\tilde{a}_i\} \le UPOWA$ ($\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n$) $\le$ Max$\{\tilde{a}_i\}$. It is idempotent because if $\tilde{a}_i = \tilde{a}$, for all $\tilde{a}_i$, then, $UPOWA$ ($\tilde{a}_1, \tilde{a}_2 \ldots, \tilde{a}_n$) $= \tilde{a}$.

4. Families of UPOWA Operators

Different types of UPOWA operators can be studied depending on the considerations made in the analysis. First of all we are going to consider the two main cases of the UPOWA operator that are found by analyzing the coefficient β. Basically, if β = 0, then, we get the probabilistic approach and if β = 1, the UOWA operator. The more of β located to the top, the more we use the UOWA operator and vice versa.

By using a different manifestation in the weighting vector W (or P) we can analyze a wide range of particular cases. For example, we can obtain the uncertain probabilistic maximum, the uncertain probabilistic minimum, the uncertain arithmetic probabilistic aggregation (UAPA), the uncertain probabilistic weighted average (UPWA) and the uncertain arithmetic OWA (UAOWA) operator.

Remark 1. The uncertain probabilistic maximum is found when $w_1 = 1$ and $w_j = 0$ for all $j \neq 1$. The probabilistic minimum is formed when $w_n = 1$ and $w_j = 0$ for all $j \neq n$.

Remark 2. More generally, the step-UPOWA is formed when $w_k = 1$ and $w_j = 0$ for all $j \neq k$. Note that if $k = 1$, the step-UPOWA is transformed into the uncertain probabilistic maximum, and if $k = n$, the step-UPOWA becomes the uncertain probabilistic minimum operator.

Remark 3. The UAPA operator is obtained when $w_j = 1/n$ for all j, and the uncertain probabilistic weighted average is obtained when the ordered position of i is the same as the ordered position of j. The UAOWA operator is obtained when $p_i = 1/n$ for all i.

Remark 4. For the median-UPOWA, if n is odd we assign $w_{(n+1)/2} = 1$ and $w_{j*} = 0$ for all others. If n is even we assign for example, $w_{n/2} = w_{(n/2)+1} = 0.5$ and $w_{j*} = 0$ for all others.

Remark 5. The olympic-UPOWA is generated when $w_1 = w_n = 0$, and for all others $w_{j*} = 1/(n-2)$. Note that it is possible to develop a general form of the olympic-UPOWA by considering that $w_j = 0$ for $j = 1, 2, \ldots, k, n, n-1, \ldots, n-k+1$, and for all others $w_{j*} = 1/(n-2k)$, where $k < n/2$. Note that if $k = 1$, then this general form becomes the usual olympic-UPOWA. If $k = (n-1)/2$, then, this general form becomes the median-UPOWA aggregation. That is, if n is odd, we assign $w_{(n+1)/2} = 1$, and $w_{j*} = 0$ for all other values. If n is even, we assign, for example, $w_{n/2} = w_{(n/2)+1} = 0.5$ and $w_{j*} = 0$ for all other values.

Remark 6. Note that it is also possible to develop the contrary case of the general olympic-UPOWA operator. In this case, $w_j = (1/2k)$ for $j = 1, 2, \ldots, k, n, n-1, \ldots, n-k+1$, and $w_j = 0$, for all other values, where $k < n/2$. Note that if $k = 1$, then we obtain the contrary case for the median-UPOWA.

Remark 7. Another interesting family is the S-UPOWA operator. It can be subdivided into three classes: the "or-like," the "and-like" and the generalized S-UPOWA operators. The generalized S-UPOWA operator is obtained if $w_1 = (1/n)(1 - (\alpha + \beta)) + \alpha$, $w_n = (1/n)(1 - (\alpha + \beta)) + \beta$, and $w_j = (1/n)(1 - (\alpha + \beta))$ for $j = 2$ to $n - 1$, where $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \le 1$. Note that if $\alpha = 0$, the generalized S-UPOWA operator becomes the "and-like" S-UPOWA operator, and if $\beta = 0$, it becomes the "or-like" S-UPOWA operator.

Remark 8. Another family of aggregation operator that could be used is the centered-UPOWA operator. We can define an UPOWA operator as a centered aggregation operator if it is symmetric, strongly decaying and inclusive. Note that these properties have to be accomplished for the weighting vector W of the UOWA operator but not necessarily for the weighting vector P of the probabilities. It is symmetric if $w_j = w_{j+n-1}$. It is strongly decaying when $i < j \leq (n + 1)/2$ then $w_i < w_j$ and when $i > j \geq (n + 1)/2$ then $w_i < w_j$. It is inclusive if $w_j > 0$. Note that it is possible to consider a softening of the second condition by using $w_i \leq w_j$ instead of $w_i < w_j$, then, we get the softly decaying centered-UPOWA operator. And if we remove the third condition, we get the non-inclusive centered-UPOWA operator.

Remark 9. A further interesting type is the non-monotonic-UPOWA operator. It is obtained when at least one of the weights w_j is lower than 0 and $\sum_{j=1}^{n} w_j = 1$. Note that a key aspect of this operator is that it does not always achieve monotonicity.

Remark 10. Other families of UPOWA operators could be used following the recent literature about different methods for obtaining OWA weights (Merigó and Gil-Lafuente 2009; Yager 1993, 2009).

5. Multi-Person Decision-Making with the UPOWA Operator

The UPOWA operator can be applied in a wide range of fields because all the previous studies that use the probability or the OWA operator can be revised and extended with this new approach. For example, we could develop a wide range of applications in statistics, economics, engineering and decision theory. In this paper, we focus on a decision-making application in the selection of national strategies by the government, such as the selection of monetary policies, using a multi-person analysis. A multi-person analysis provides a more complete representation of the problem because it is based on the opinion of several people. Therefore, we can aggregate the opinion of different people to obtain a representative view of the problem. In politics and national decision-making, this is very useful because usually decisions are not individual, but are made by a group of people in the parliament or in the ministries council.

The procedure to select monetary policies with the UPOWA operator in multi-person decision-making is described in this section. Many other group decision-making models have been discussed in the literature (Merigó and Casanovas 2011b; Wei *et al.* 2010; Xu 2010).

Step 1: Let $A = \{A_1, A_2, \ldots, A_m\}$ be a set of finite alternatives, $S = \{S_1, S_2, \ldots, S_n\}$, a set of finite states of nature (or attributes), forming the payoff matrix $(\tilde{a}_{hi})_{m \times n}$. Let $E = \{e_1, e_2, \ldots, e_q\}$ be a finite set of decision-makers. Let $U = (u_1, u_2, \ldots, u_p)$ be the weighting vector of the decision-makers such that $\sum_{k=1}^{q} u_k = 1$ and $u_k \in [0, 1]$. Each decision-maker provides his own payoff matrix $(\tilde{a}_{hi}{}^{(k)})_{m \times n}$.

Step 2: Calculate the weighting vector $\hat{P} = \beta \times W + (1 - \beta) \times P$ to be used in the UPOWA aggregation. Note that $W = (w_1, w_2, \ldots, w_n)$ such that $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0, 1]$ and $P = (p_1, p_2, \ldots, p_p)$ such that $\sum_{i=1}^{n} p_i = 1$ and $p_i \in [0, 1]$.

Step 3: Use the WA to aggregate the information of the decision-makers E using the weighting vector U. The result is the collective payoff matrix $(\tilde{a}_{hi})_{m \times n}$. Thus, $\tilde{a}_{hi} = \sum_{k=1}^{p} u_k \tilde{a}_{hi}^{k}$. Note that it is possible to use other types of aggregation operators instead of the WA to aggregate this information.

Step 4: Calculate the aggregated results using the UPOWA operator explained in Eq. (5). Consider different families of UPOWA operators as described in Section 4.

Step 5: Adopt decisions according to the results found in the previous steps. Select the alternative (s) that provides the best result (s). Moreover, establish an ordering or a ranking of the alternatives from the most- to the least-preferred alternative, enabling consideration of more than one selection.

This aggregation process can be summarized using the following aggregation operator that we call the multi-person – UPOWA (MP-UPOWA) operator.

Definition 6. A MP-UPOWA operator is a mapping $MP\text{-}UPOWA: \Omega^n \times \Omega^p \rightarrow \Omega$ that has a weighting vector U of dimension p with $\sum_{k=1}^{p} u_p = 1$ and $u_k \in [0, 1]$ and a weighting vector W of dimension n with $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0, 1]$, such that:

$$MP\text{-}UPOWA\ ((\tilde{a}_1^{\ 1}, \ldots, \tilde{a}_1^{\ p}), \ldots, (\tilde{a}_n^{\ 1}, \ldots, \tilde{a}_n^{\ p})) = \sum_{j=1}^{n} \hat{p}_j b_j, \quad (9)$$

where b_j is the jth largest of the $\tilde{a}_i$, each argument $\tilde{a}_i$ is an interval number and has an associated weight (WA) v_i with $\sum_{i=1}^{n} p_i = 1$ and $p_i \in [0, 1]$, $\hat{p}_j = \beta w_j + (1-\beta) p_j$ with $\beta \in [0, 1]$ and p_j is the probability p_i ordered according to b_j, that is, according to the jth largest of the $\tilde{a}_i$, $\tilde{a}_i = \sum_{k=1}^{p} u_k \tilde{a}_i^k$, $\tilde{a}_i^k$ is the argument variable provided by each person (or experts).

Note that the MP-UPOWA operator has similar properties than those explained in Section 3, such as the distinction between descending and ascending orders, the aggregation with uncertain weights, and so on.

The MP-UPOWA operator includes a wide range of particular cases following the methodology explained in Section 4. Thus, it includes the multi-person – UPA (MP-UPA) operator, the multi-person – UOWA (MP-UOWA) operator, the multi-person – uncertain average (MP-UA) operator, the multi-person – arithmetic-UPA (MP-AUPA) operator and the multi-person – arithmetic-UOWA (MP-AUOWA) operator.

6. Illustrative Example

In the following, we present a numerical example of the new approach in a multi-person decision-making problem regarding the selection of national strategies. We analyze an economic problem concerning the selection of the optimal monetary policy of a country. Note that other decision-making applications could be developed in other areas such as construction design (Turskis *et al.* 2009) and the selection of project managers (Zavadskas *et al.* 2010b).

Step 1: Assume the government of a country has to decide on the type of monetary policy to use the next year. They consider six alternatives:

A_1 = Develop an extremely strong expansive monetary policy.
A_2 = Develop a strong expansive monetary policy.
A_3 = Develop an expansive monetary policy.
A_4 = Do not develop any change in the monetary policy.
A_5 = Develop a contractive monetary policy.
A_6 = Develop a strong contractive monetary policy.

In order to evaluate these strategies, the government has brought together a group of experts. This group considers that the key factor is the economic situation of the world economy for the next period. They consider 7 possible states of nature that could happen in the future:

S_1 = Very bad economic situation.

S_2 = Bad economic situation.

S_3 = Regular – Bad economic situation.

S_4 = Regular economic situation.

S_5 = Regular – Good economic situation.

S_6 = Good economic situation.

S_7 = Very good economic situation.

The experts are classified in 3 groups. Each group is led by one expert and gives different opinions than the other two groups. The results of the available strategies, depending on the state of nature S_i and the alternative A_k that the government chooses, are shown in Tables 1, 2 and 3.

Table 1. Opinion of the first group of experts

	S_1	S_2	S_3	S_4	S_5	S_6	S_7
A_1	(40,50,60)	(30,40,50)	(60,70,80)	(70,80,90)	(80,90,100)	(60,70,80)	(70,80,90)
A_2	(60,70,80)	(70,80,90)	(50,60,70)	(40,50,60)	(60,70,80)	(50,60,70)	(60,70,80)
A_3	(10,20,30)	(50,60,70)	(70,80,90)	(80,90,100)	(30,40,50)	(70,80,90)	(40,50,60)
A_4	(80,90,100)	(60,70,80)	(40,50,60)	(60,70,80)	(20,30,40)	(40,50,60)	(70,80,90)
A_5	(40,50,60)	(10,20,30)	(60,70,80)	(20,30,40)	(30,40,50)	(60,70,80)	(50,60,70)
A_6	(10,20,30)	(20,30,40)	(40,50,60)	(70,80,90)	(30,40,50)	(60,70,80)	(40,50,60)

Table 2. Opinion of the second group of experts

	S_1	S_2	S_3	S_4	S_5	S_6	S_7
A_1	(50,60,70)	(30,40,50)	(40,50,60)	(60,70,80)	(70,80,90)	(50,60,70)	(60,70,80)
A_2	(70,80,90)	(70,80,90)	(30,40,50)	(40,50,60)	(60,70,80)	(50,60,70)	(40,50,60)
A_3	(30,40,50)	(80,90,100)	(70,80,90)	(60,70,80)	(30,40,50)	(70,80,90)	(50,60,70)
A_4	(60,70,80)	(30,40,50)	(50,60,70)	(60,70,80)	(20,30,40)	(60,70,80)	(40,50,60)
A_5	(40,50,60)	(40,50,60)	(80,90,100)	(60,70,80)	(40,50,60)	(70,80,90)	(50,60,70)
A_6	(20,30,40)	(20,30,40)	(30,40,50)	(60,70,80)	(50,60,70)	(70,80,90)	(40,50,60)

Table 3. Opinion of the third group of experts

	S_1	S_2	S_3	S_4	S_5	S_6	S_7
A_1	(60,70,80)	(30,40,50)	(40,50,60)	(60,70,80)	(70,80,90)	(80,90,100)	(40,50,60)
A_2	(40,50,60)	(70,80,90)	(60,70,80)	(50,60,70)	(40,50,60)	(70,80,90)	(40,50,60)
A_3	(30,40,50)	(60,70,80)	(70,80,90)	(70,80,90)	(30,40,50)	(70,80,90)	(50,60,70)
A_4	(60,70,80)	(20,30,40)	(70,80,90)	(60,70,80)	(30,40,50)	(60,70,80)	(30,40,50)
A_5	(40,50,60)	(30,40,50)	(70,80,90)	(60,70,80)	(40,50,60)	(30,40,50)	(50,60,70)
A_6	(30,40,50)	(20,30,40)	(40,50,60)	(40,50,60)	(40,50,60)	(70,80,90)	(60,70,80)

Step 2–3: In this problem, we assume the following weighting vector for the three group of experts: $U = (0.3, 0.3, 0.4)$. The experts assume the following weighting vector for the OWA: $W = (0.1, 0.1, 0.1, 0.1, 0.2, 0.2, 0.2)$; for the probability: $P = (0.1, 0.1, 0.2, 0.2, 0.2, 0.1, 0.1)$; and $\beta = 40\%$. Thus, with these OWA weights we see that they are assuming a pessimistic attitude because they give more importance to the worst results. On the other hand, we see with the probabilities that they believe that the economic situation for the next year will be moderate as they give more importance to the central part of the probabilistic weights. First, we aggregate the information of the three groups into one collective matrix that represents all the experts of the problem. The results are shown in Table 4.

Table 4. Collective results

	S_1	S_2	S_3	S_4	S_5	S_6	S_7
A_1	(51,61,71)	(30,40,50)	(46,56,66)	(63,73,83)	(73,83,93)	(65,75,85)	(55,65,75)
A_2	(55,65,75)	(70,80,90)	(48,58,68)	(44,54,64)	(52,62,72)	(58,68,78)	(46,56,66)
A_3	(24,34,44)	(63,73,83)	(70,80,90)	(70,80,90)	(30,40,50)	(70,80,90)	(47,57,67)
A_4	(66,76,86)	(35,45,55)	(55,65,75)	(60,70,80)	(24,34,44)	(54,64,74)	(45,55,65)
A_5	(40,50,60)	(27,37,47)	(70,80,90)	(48,58,68)	(37,47,57)	(51,61,71)	(50,60,70)
A_6	(21,31,41)	(20,30,40)	(37,47,57)	(55,65,75)	(40,50,60)	(67,77,87)	(48,58,68)

Step 4: With this information, we can aggregate the expected results for each state of nature in order to make a decision by using Eq. (5). In Table 5, we present the results obtained using different types of UPOWA operators. Note that we can also obtain these results by using Eq. (4). Obviously, we get the same results with both methods.

Table 5. Aggregated results

	Max-UPA	**Min-UPA**	**UA**	**UPA**	**UOWA**	**UPOWA**
A_1	(63.1,73.1,83.1)	(45.9,55.9,65.9)	(54.7,64.7,74.7)	(56.5,66.5,76.5)	(51,61,71)	(54.3,64.3,74.3)
A_2	(59.0,69.0,79.0)	(48.6,58.6,68.6)	(53.2,63.2,73.2)	(51.7,61.7,71.7)	(51.1,61.1,71.1)	(51.4,61.4,71.4)
A_3	(60.6,70.6,80.6)	(42.2,52.2,62.2)	(53.4,63.4,73.4)	(54.4,64.4,74.4)	(47.5,57.5,67.5)	(51.6,61.6,71.6)
A_4	(55.0,65.0,75.0)	(38.2,48.2,58.2)	(48.4,58.4,68.4)	(47.8,57.8,67.8)	(44.3,54.3,64.3)	(46.4,56.4,66.4)
A_5	(56.6,66.6,76.6)	(39.4,49.4,59.4)	(46.1,56.1,66.1)	(47.8,57.8,67.8)	(42.7,52.7,62.7)	(45.7,55.7,65.7)
A_6	(52,62,72)	(33.2,43.2,53.2)	(41.1,51.1,61.1)	(42,52,62)	(36.6,46.6,56.6)	(39.8,49.8,59.8)

Step 5: If we establish an ordering of the alternatives, then we get the results shown in Table 6. Note that the first alternative in each ordering is the optimal choice.

Table 6. Ranking of the monetary policies

	Ordering		**Ordering**
Max-UPA	$A_4\}A_5\}A_3\}A_2\}A_1\}A_6$	UPA	$A_5\}A_1\}A_2\}A_4\}A_3\}A_6$
Min-UPA	$A_1\}A_5\}A_2\}A_4\}A_3\}A_6$	UOWA	$A_5\}A_1\}A_2\}A_4\}A_3\}A_6$
UA	$A_5\}A_1\}A_2\}A_4\}A_3\}A_6$	UPOWA	$A_5\}A_1\}A_2\}A_4\}A_3\}A_6$

Evidently, the order preference for the monetary policy strategies may be different, depending on the aggregation operator used. Therefore, the decision about which strategy to select may be also different. However, in this example it seems that A_5 should be the optimal choice excepting for some extreme pessimistic situations where A_1 would be the optimal one.

7. Conclusions

We have presented the UPOWA operator. It is an aggregation operator that unifies the OWA operator and the probability in the same formulation and in an uncertain environment that can be assessed with interval numbers. The main advantage of this new model is that it is able to unify the probability and the OWA operator giving different degrees of importance to them according to the relevance they have in the specific problem considered. Moreover, by using interval numbers, we are able to provide more complete information to the decision maker because we represent the environment considering the best and worst result that could occur under uncertainty. We have compared this approach with the concept of immediate probability and we have seen how the UPOWA operator is able to overcome the main limitations of the immediate probability by considering the degree of importance that the probability and the OWA operator has in the aggregation. We have also studied some of its main properties and particular cases including the uncertain probabilistic minimum, the uncertain probabilistic maximum, the UOWA, the UPA, the UAOWA and the UAPA operator.

We have also studied the applicability of this new approach and we have seen that it is very broad because all the studies that use the probability or the OWA operator can be revised and extended with this new approach. The reason is that we can always reduce this new approach to the classical cases where we only use the probability or the OWA operator. We have seen that it is possible to apply it in statistics, economics and engineering. We have focussed on an application in a multi-person decision making problem. Thus, we have obtained the MP-UPOWA operator that permits to consider the opinion of several experts in the analysis. It also includes a wide range of particular cases including the MP-UPA and the MP-UOWA operator. We have developed an example in a national decision-making problem concerning policy management. We have analyzed the selection of monetary policies in a country.

In future research, we expect to develop further extensions of the UPOWA operator by using other techniques for representing the uncertainty (fuzzy numbers, linguistic variables, etc.) and other variables such as order inducing variables, generalized means, distance measures and more complex structures. We will also consider other applications giving special attention to statistics and decision theory such as the development of a new variance and covariance measure with the UPOWA operator and the development of a new linear regression model.

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TIKIMYBINIAI SUMAVIMO OPERATORIAI IR JŲ TAIKYMAS PRIIMANT GRUPINIUS SPRENDIMUS NEAPIBRĖŽTOJE APLINKOJE

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Santrauka. Autoriai pristato tikimybinį svertinio vidurkio operatorių, taikytiną neapibrėžtumo sąlygomis. Tai tikimybėmis pagrįstas sumavimo operatorius, kuris kartu su svertinio vidurkio operatoriais gali įvertinti alternatyvų svarbumo laipsnį. Be to, jis gali operuoti neapibrėžta informacija, išreikšta skaičiais intervaluose. Pagrindinis šio operatoriaus privalumas yra tas, kad jį galima taikyti uždaviniams, kuriuose informacija yra netiksli. Išnagrinėtos kai kurios minėto operatoriaus savybės. Sukurtas metodas pritaikytas monetarinei politikai parinkti, situacijai, kai sprendimus priima žmonių grupė. Modelis suteikia išsamesnę informaciją apie problemą, nes gali įvertinti neapibrėžtumus ir riziką.

Reikšminiai žodžiai: operatorius, tikimybės, neapibrėžtumas, skaičių intervalas, grupinis sprendimų priėmimas, politika.

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