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SHAPE DETERMINING OF A LOADED CABLE VIA TOTAL DISPLACEMENTS

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Abstract. Cable structures are very efficient (in economic aspect) when applied to cover large spans. The cable structure consists of a single cable or a network of cables. The cable attractive feature is the highest ratio of strength to weight amongst other carrying structural elements, usually applied in engineering practice. But a cable is a specific structural element able to response only one type of deforming – tension (flexural rigidity actually vanishes). Therefore, when loaded a cable shapes the form to resist tension only. This adaptation is followed by large nonlinear displacements. Thus, the nature of geometrical nonlinear cable behavior is quitie a different from that of rigid structural elements. Both elements response via small deformations when loaded, but large displacements of a cable are conditioned by its adaptation to loading, and those of rigid structural elements – by actual deformations. One can also note that deformations of a cable are significantly less than those of rigid structural elements, but at the same time actual cable displacements are significantly larger. Thus, the main disadvantage of a cable structure is its response to loading by large displacements caused by asymmetric loading component (usually met in engineering practice, e.g. the design of suspension bridges, coverings of stadium, etc). Therefore stiffness conditions predominate in the actual codified cable design. Having identified governing factors conditioning displacement magnitudes one can introduce the constructional means/solutions assigned to reduce them if required. Therefore the evaluation of cable displacements by a reliable and sufficiently exact method compatible with the calculation of actual engineering structures is under current necessity. When analyzing total displacements the principle of superposition is employed in a special sequence. Total displacement is split into two components: kinematic and elastic. The first component represents cable form shaping the loading, the second one is conditioned by elastic deformations. Any point displacement of an asymmetrically loaded cable can be expressed via its middle span. The developed analytical expressions to evaluate middle span displacements are presented. They enable to identify maximal displacements and their locations. The developed analytical method for total displacements evaluation is tested numerically. The comparative analysis in respect of the influence of various parameters conditioning displacement magnitudes is performed. The displacement evaluation errors, their causality conditioned by the application of approximate- widely applied engineering methods, are discussed.

Keywords: cable structure shape, nonlinear analysis, asymmetric loading, total displacements

1. Introduction

Cable or combined structures containing a single cable or cable networks are widely employed as the main carrying elements in engineering practice. It is conditioned by an attractive combination: small weight and high strength (the highest ratio of strength to weight) when compared with other usually applied carrying structural elements. Small weight is one of the governing factors to choose the carrying structure for covering large spans (e g bridges) or large spaces (e. g. stadiums) [1–12]. However, the excellent carrying strength of a cable is accompanied by an essential disadvantage – it responses to asymmetric loading via large displacements. This is conditioned by a specific feature of a cable – it can resist only one type of deforming

– tension (flexural rigidity actually vanishes). Therefore the adaptation to loading is followed by certain shape changes to fit the loading. Analyzing cable shape changes, one must note that the main contribution to developed total displacements is related to the ones, ensuring adaptation to asymmetric loading. These displacements are denoted as kinematic components of total displacements or simply – kinematic displacements. The other displacement components accumulated by actual deformations are significantly less kinematic ones. This is conditioned by cable material properties, where high elastic strength is correlated to high elasticity modulus. One must mark that cable response to loading via kinematic displacements is strongly nonlinear, i.e. geometrical nonlinearity of specific nature is noticed. Therefore cable reaction to loading by total displacements is also geometrically nonlinear.

The main disadvantage of a loaded cable structure is developed large displacements conditioned mainly by kinematic displacements ([3, 4, 8–11, 13, 14]). One must note that the distribution of internal forces in a cable generally depends on its shape, the elastic displacements can be defined only having accurately evaluated the kinematic ones. Design and analysis of cable structures was investigated in many works ([1, 3, 8, 14–17]). Most of them are assigned to evaluation of cable total displacements ([3, 16–20]), these neglecting a qualitative analysis of a confounding factor, i.e. the influence of kinematic displacements on the actual shape of a cable and its influence to actual stress and strain state.

The stiffness requirements in concert with the strength ones are introduced in design codes. They state that generally extreme (usually maximal vertical) displacement due to all considered load combinations $i = 1, \ ..., \eta$ can not exceed the prescribed magnitude $\omega_{\lim}$, i.e. $\omega_{\max,i} \leq \omega_{\lim}$ for all $i = 1, \ ..., \eta$.

Therefore identifying a governing factor for cable displacement magnitude is the reliable estimation of kinematic displacements conditioning correct evaluation of total displacements (elastic ones are prescribed by the latter ones) is very important. One can mention the authors who recommend only to analyze the kinematic displacements, mostly influencing the cable shape ([8, 10, 13, 15, 18]). Many investigations on the evaluation of kinematic displacements are assigned to so called engineering methods employing the superposition principle when splitting the actual loads to symmetric and asymmetric components ([10, 13, 15]). Such rather simplified approach, in some cases yields an inadmissible error valuating the shape of a suspension cable, responding to loading in a geometrically nonlinear way of specific nature. The investigation [14] indicates the errors conditioned by replacing of actual asymmetric loading to symmetric and asymmetric components. Aiming to uprate the accuracy of engineering methods, the equivalent correcting loading, is proposed to introduce ([8, 21]). One must mark that such approach seems to be rather simplified and artificial, furthermore no any clear definition or algorithm to identify the equivalent loading is presented. Note that one can find only few ([8, 22]) investigations, assigned on corrected evaluation of kinematic displacements. However, a more exhaustive cable kinematical behavior analysis should be useful in practical aspect: that evaluating the actual shape and stress state of a loaded cable; that determining the cases/bounds for admissible application of engineering methods.

The evaluation of cable shape by employing general analytical expressions is rather complicated, as requires many computational efforts during iterative problem solution procedures ([3, 8, 16, 17]). Therefore the direct employment of above expressions for practical calculations/design is under question. Thus, the creation of the method allowing the evaluation of cable shape via total (kinematic and elastic) displacements with sufficient accuracy and relatively smaller computational efforts, is under actual necessity.

The paper is assigned to the shape analysis of an asymmetrically loaded suspension cable via total displacements by splitting them to kinematic and elastic components. Kinematical displacements are analyzed for cable loaded and unloaded parts. The developed analytical expressions for their determining are presented. A method of evaluating elastic displace- ments (conditioned by the kinematic ones) and subsequently the total ones is presented. The obtained analytical expressions are illustrated by numerical applications. The evaluation of errors conditioned by the application of engineering methods is discussed.

2. Estimating of kinematic displacements

2.1. Vertical kinematic displacements

It is known that the equilibrium shape (form) of a cable when subjected by symmetrical (uniformly distributed per cable span) loading (e.g. that of cable weight force or that of analogous applied additionally) fits quadratic parabola. Such loading causes only elastic cable displacements with maximal one at cable middle span ([3, 8, 14, 17]). The supplemental uniformly distributed load applied onto the cable part from support till middle span (a typical case) enables the cable to change the primary shape to a new one (due to a new moment diagram), i.e. the cable adapts to respond the loading via tensile resistance. This process is accompanied by large displacements. Seeking to obtain pure kinematic displacements one must eliminate the contribution of elastic displacements. It can be done introducing infinitesimally large cable axial stiffness $EA \to \infty$, where A is cable cross-sectional area, E is material elasticity modulus. In this case elastic displacements vanish. Practically it can be done introducing a sufficiently large e.g. elasticity modulus, resulting in the elastic displacements of magnitudes available to neglect in the analysis of total displacements. Thus, the cable primary shape change i.e. developed displacements are conditioned by kinematic ones only. Due to widely applied engineering methods ([10, 13, 15]) maximal displacements develop in both (subjected to supplemental loading and free from it) cable parts. They are assumed to be equal in absolute magnitudes, resulting the middle span displacement to be zero. Such an approach allows obtaining simpler analytical solutions, but certainly conditions the errors when determining actual cable kinematic displacements.

Consider the behavior of a suspension cable subjected to the following loading: uniformly distributed load q, applied per total span (symmetric load); uniformly distributed load p, applied onto left half span $l/2$ (asymmetric load) (see Fig 1).

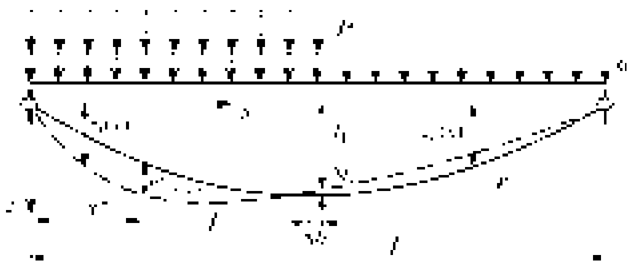

Fig 1. Cable shape change due to an asymmetric load

The primary shape (cable axis) corresponds to quadratic parabola function:

$$z(x) = M_0(x) \Big/ H_0 = f_0 \left(4x \big/ l - 4x^2 \big/ l^2 \right), \qquad (1)$$

here $M_0(x)$ is the moment caused by symmetric load q; H_0 is a thrusting (tensile) inner cable force.

The axis of a displaced cable (due to kinematic displacements) is described by the following functions different for cable left $z_{lk}(x)$ and right $z_{rk}(x)$ parts ([23]), namely:

$$z_{lk}(x) = f_{k1} \Big/ \left(1 + \gamma \big/ 2\right) \begin{bmatrix} \left(4x \big/ l - 4x^2 \big/ l^2 \right) + \\ \gamma \left(4x \big/ l - 4x^2 \big/ l^2 \right) \end{bmatrix} \qquad (2)$$

for $x \le l/2$, and

$$z_{lk}(x) = f_{k1} \Big/ \left(1 + \gamma \big/ 2\right) \begin{bmatrix} \left(4x \big/ l - 4x^2 \big/ l^2 \right) + \\ \gamma \left(x \big/ l - 1 \right) \end{bmatrix} \qquad (3)$$

for $l/2 \le x \le l$,

here

f_{k1} is cable sag of kinematic nature (kinematic sag) at cable middle span fixed after applying supplemental loading in concert with symmetric one q;

$\gamma = p/q$ is ratio of symmetric and asymmetric loads (ratio of their intensities).

The cable kinematic sag at middle span f_{k1} can be expressed via primary cable sag f_0 (the middle span vertical displacement of cable subjected by symmetric load p), namely:

$$f_{k1} = f_0 + \Delta f_k \,. \qquad (4)$$

Analyzing the relations (2) and (3) one can find that the function describing the cable left part axis is the sum of two parabola functions, while the one of the right part corresponds to the sum of parabola and line functions.

The location of maximal displacement (deflection) can be identified by equaling the first derivative of function to zero, i.e. $z'_{lk}(x) = 0$ results:

$$x = \frac{l}{4} \cdot \frac{(2 + 3\gamma/2)}{(1+\gamma)} \,. \qquad (5)$$

The position of maximal deflection depends on loads ratio $\gamma = p/q$, as one can find from expression (5). The variation of γ in the bounds from 1 to 10 results an adequate variation the position of maximal deflection inside the bounds fixed by $x^* = 0.437l \div 0.386\, l$.

One can find that the determination of any point kinematic sag of an asymmetrically loaded cable is rather simple applying the formulae (2) and (3), but the cable middle span sag f_{k1} must be known. To obtain the latter value one must apply a full set of equations (i.e. statical, geometrical an physical ones) ([3, 8, 10, 14, 17]).

The cable primary length according to the well-known formula is ([14]):

$$s_0 \cong l + \frac{8}{3} \frac{f_0^2}{l} \,. \qquad (6)$$

The cable length can be determined as the sum of cable left and right parts fixed for changed shape by:

$$s_{1k} = s_{lk} + s_{rk} \cong l + 0.5 \int_0^{l/z} \left[z'_{lk}(x) \right]^2 dx + 0.5 \int_{l/2}^{l} \left[z'_{rk}(x) \right]^2 dx. \qquad (7)$$

The solution of (7), combining (2) and (3), finally yields:

$$s_{1k} \cong l + \frac{8}{3}\frac{f_{k1}^2}{l}\left[\frac{1+\gamma+5\gamma^2/16}{1+\gamma+\gamma^2/4}\right]. \qquad (8)$$

When the cable shape changes are conditioned only by kinematic displacements $s_0 = s_{1k}$ then the cable kinematic sag is calculated by:

$$f_{k1} = f_0 \frac{(1+\gamma/2)}{\sqrt{1+\gamma+5\gamma^2/16}}. \qquad (9)$$

The formula (9) results the cable kinematic sag at middle span f_{k1} to be less the primary one f_0 in the case when $\gamma \neq 0$. Thus, the kinematic displacement at middle span Δf_k is of a negative sign (i.e. it is directed up, in opposite to loading direction). The kinematic displacement Δf_k magnitude, taking into account (4) can be calculated by

$$\Delta f_k = -f_0\left(1-\frac{1+\gamma/2}{\xi}\right), \qquad (10)$$

where $\xi = \sqrt{1+\gamma+5\gamma^2/16}$.

The expression (10) proves that kinematic displacement Δf_k is always directed up for any positive γ. This is evidently illustrated via the graph, presented in Fig 2. The graph is simulated for a suspension cable of span $l = 100\ m$ and primary sags $f_0 = 10\ m$ and $f_0 = 20\ m$.

The increment of load ratio γ is compatible with the increment of negative (directed up) middle span vertical displacement Δf_k. This relationship is nonlinear one, as one can find from Fig 2. Find that kinematic displacement at middle span Δf_k is equal to zero only when $\gamma = 0$. Besides, applying the statical equations, one can obtain the relationship of thrusting force H_{k1} vs kinematic displacement Δf_k, reading

$$H_{k1} = \frac{ql^2(1+\gamma/2)}{8(f_0+\Delta f_k)}, \qquad (11)$$

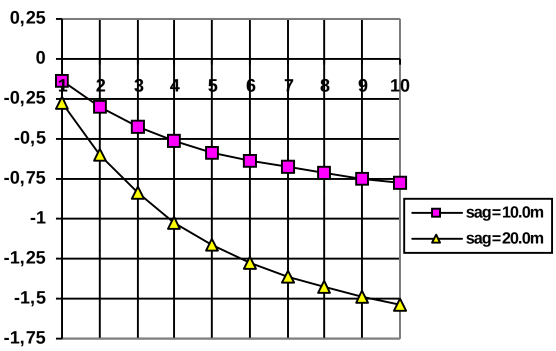

Fig 2. Cable middle span vertical displacement (in *m*) versus load ratio γ

On the basis of the analysis of formulae (10) and (11), one can state that the application of the engineering method (assuming the middle span vertical displacement of an asymmetrically loaded cable to be $\Delta f_k = 0$), results in errors determining the actual cable shape.

2.2. Horizontal kinematic displacements

Kinematic horizontal displacements develop simultaneously with vertical kinematic ones ([8, 9, 14]). They are directed to a cable part loaded by asymmetric load *p*. In other words, one can say that the left cable part contribution to total cable length increases and that of the right part – decreases. Taking into account this feature, one can determine both, i.e. middle span kinematic displacements of left and right cable parts, namely:

$$\Delta h_{lk} = (s_{lk} - s_{l0})\cos\varphi_x \qquad (12)$$

and

$$\Delta h_{rk} = (s_{rk} - s_{r0})\cos\varphi_x, \qquad (13)$$

where φ_x is angle between cable slope and horizontal axis.

Assuming $\varphi_x \approx 0$ at a cable middle span and employing (6), (7) and (8) one can obtain the formulae to determine horizontal kinematic displacements:

$$\Delta h_{lk} = \frac{4}{3l}\left[(f_0+\Delta f_k)^2\frac{\left(1+5\gamma/4+7\gamma^2/16\right)}{(1+\gamma/2)^2} - f_0^2\right], \qquad (14)$$

$$\Delta h_{rk} = \frac{4}{3l}\left[(f_0+\Delta f_k)^2\frac{\left(1+3\gamma/4+3\gamma^2/16\right)}{(1+\gamma/2)^2} - f_0^2\right]. \qquad (15)$$

The analysis of the above formulae shows that horizontal kinematic displacements as well as vertical ones depend on cable primary sag f_0 and loads ratio γ. The graphs of middle span horizontal displacements vs γ are illustrated for cable of $l = 100\ m$ and $f_0 = 10\ m$ in Fig 3.

Besides, when analyzing the formulae (14) and (15), one can find that middle span kinematic displacements are related to kinematic vertical ones. Numerical simulations proved the horizontal kinematic displacements Δh_{lk} and Δh_{rk} to be the magnitudes of the same order as Δf_k. Also one can find that horizontal kinematic displacements of cable left and right parts are equal in absolute values, i.e. $|\Delta h_l| = |\Delta h_r|$.

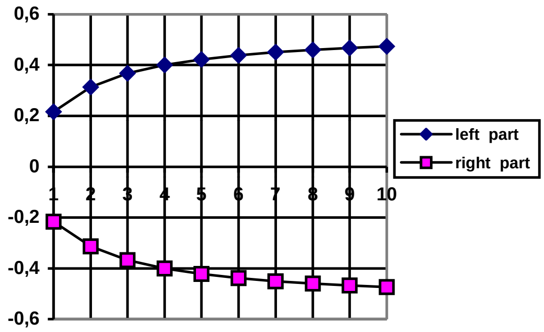

Fig 3. Horizontal middle span kinematic displacements of cable ($l = 100$ m, $f_0 = 10$ m) left and right parts (in *m*) vs loads ratio γ

One can note that middle span kinematic displacements prescribe the shape of an asymmetrically loaded cable as well as its deformed state. It is evident that as efficient tools, reducing maximal vertical displacements, constructive means can serve to constraint horizontal kinematic displacements.

3. Maximal cable kinematic displacements

The codified design aims to evaluate maximal vertical displacements, as it was mentioned above, to check whether stiffness conditions are not violated. Thus, pure ($EA \to \infty$) maximal vertical kinematic displacements, as governing ones when conditioning loaded cable shape, are to be determined.

3.1. Cable left part displacements

The vertical kinematic displacements of the left loaded by *p* cable part, can be obtained as the difference of deformed and primary cable shapes ([8, 14, 22]):

$$\omega_{lk}(x) = z_{lk}(x) - z_{l0}(x), \tag{16}$$

where

$z_{l0}(x)$ is cable left part primary shape function;

$z_{lk}(x)$ is cable left part shape function after its loading by p.

Combining (1) and (2) relations with (16) one can obtain the formula for determining vertical kinematic displacement of cable left part $(x \le l/2)$:

$$\omega_{lk}(x) = \frac{f_0 + \Delta f_k}{(1+\gamma/2)}\left[\left(\frac{4x}{l} - \frac{4x^2}{l^2}\right) + \gamma\left(\frac{3x}{l} - \frac{4x^2}{l^2}\right)\right] - f_0\left(\frac{4x}{l} - \frac{4x^2}{l^2}\right). \tag{17}$$

When analyzing formula (17), one can find cable left part displacements to be dependant on primary sag f_0, loads ratio γ and cable middle span kinematic displacement Δf_k magnitudes.

Location of maximal kinematic displacement ω_{lk} can be defined when equaling the first derivative of the displacement expression (17) to zero, i.e. $\omega'_{lk}(x) = 0$ ([22]):

$$x^* = \frac{l}{4}\frac{(2+3\gamma/2-2\xi)}{(1+\gamma-\xi)}. \tag{18}$$

Analyzing (18) one can find the maximal left part displacement to be departed from the left support by distance $x^* < l/4$. Numerical simulations resulted $x^* = 0.957\ l/4$, when $\gamma = 1$; $x^* = 0.921\ l/4$, when $\gamma = 3$.

Maximal kinematic displacement of cable left part is obtained combining (18) and (10). It reads:

$$\omega_{lk,\max} = f_0\left[\left(2\beta - \beta^2\right)\left(\frac{1}{\xi} - 1\right) + \frac{\gamma}{2\xi}\left(3\beta - 2\beta^2\right)\right], \tag{19}$$

where $\beta = \dfrac{1-\xi+3\gamma/4}{1-\xi+\gamma}$.

Taking $x^* = l/4$ one can obtain an approximate formula for determining the left part displacement ([23]):

$$\omega_{lk}(x) = \frac{3}{4} f_0\left[\frac{(1+2\gamma/3)}{\xi} - 1\right]. \tag{20}$$

Find that the latter formula is quite simple one not requiring significant computational efforts. It yields an insignificant error (comparing with (19) and is less 1.6 % (for instance, when $\gamma = 10$ – it is 0.14 %; when $\gamma = 1$ – it is 1.56 %).

The relations of maximal cable left part kinematical displacements $\omega_{lk,\max}$ vs loads ratio γ are presented graphically in Fig 4 for two different primary sags.

The graph shows them to be nonlinear ones. For primary sag $f_0 = l/10 = 10.0$: $\omega_{lk,\max} = 0.721$ m when

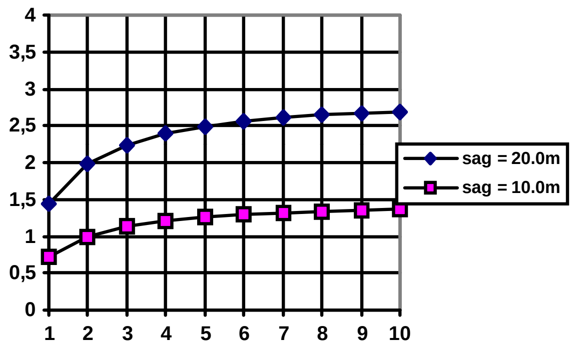

Fig 4. Cable left part maximal kinematic displacements $\omega_{lk,\max}$ vs versus loads ratio γ

$\gamma = 1$; and $\omega_{lk,\max} = 1.367$ m when $\gamma = 10$. For primary sag $f_0 = l/5 = 20.0$: $\omega_{lk,\max} = 1.443$ m when $\gamma = 1$; and $\omega_{lk,\max} = 2.735$ m when $\gamma = 10$.

It is evident that aiming to reduce the maximal vertical displacements under constant loads ratio γ, one must reduce primary sag f_0. But one must keep in mind that this results in the enlargement of cable thrusting force H_{k1} (see (11)).

3.2. Cable right part displacements

The right (unloaded) cable part displacements ω_{rk} are always negative – directed up for $\gamma > 0$. They can be determined in an analogous way as these of the cable left part. Having replaced index l by index *r* in (16) one obtains:

$$\omega_{rk}(x) = z_{rk(x)} - z_{r0}(x), \qquad (21)$$

where

$z_{r0}(x)$ is cable right part primary shape function;

$z_{rk}(x)$ is cable right part shape function after its loading by .

Combining (1) and (2) relations with (16) one can obtain the formula for determining cable right part $(x \geq l/2)$ vertical kinematic displacement:

$$\omega_{rk}(x) = \frac{f_0 + \Delta f_k}{(1 + \gamma/2)} \left[\left(\frac{4x}{l} - \frac{4x^2}{l^2} \right) + \gamma \left(1 - \frac{x}{l} \right) \right] - f_0 \left(\frac{4x}{l} - \frac{4x^2}{l^2} \right). \qquad (22)$$

The analysis of (22) shows that the right unloaded cable part maximal displacement depends on the same factors as the loaded cable left part, namely: primary sag f_0, loads ratio γ and cable middle span kinematic displacement Δf_k.

Maximal right part kinematic displacement is located at

$$x^{**} = \frac{3l}{4} \frac{(\xi - \gamma/4 - 1)}{(\xi - 1)}. \qquad (23)$$

Formula (23) shows that the variation of maximal displacement is not sensitive to different loads ratio γ magnitudes and can be approximated by constant magnitude $\omega_{rk,\max} \cong \omega_{rk}(x = 3l/4)$. The evaluation of maximal displacement according to this location does not result in the significant error, not exceeding 2.0%. Therefore the maximal displacement of an unloaded part can be determined via the formula analogous to (20). It reads:

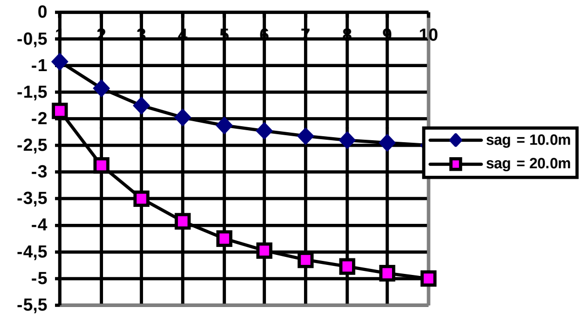

Fig 5. Cable right part maximal kinematic displacements $\omega_{rk,\max}$ vs versus loads ratio γ

$$\omega_{rk,\max}(x) = \frac{3}{4} f_0 \left[\left(\frac{1}{\xi} - 1 \right) + \frac{\gamma}{3\xi} \right]. \qquad (24)$$

The relations of maximal cable right part kinematical displacements $\omega_{lk,\max}$ vs loads ratio γ are presented graphically in Fig 5. Numerical magnitudes are presented for constant cable span $l = 100\ m$ in cases of two primary sags, namely $f_0 = 10\ m$ and $f_0 = 20\ m$ (analogous as of cable analyzed in section 3.1 already).

Analyzing the graphs of Fig 5 one can find that an increment of loads ratio γ results in the increment (in absolute values) of $\omega_{rk,\max}$. For primary sag $f_0 = l/10 = 10.0$: $\omega_{rk,\max} = -0.924\ m$ when $\gamma = 1$; and $\omega_{rk,\max} = -2.500\ m$ when $\gamma = 10$. For primary sag $f_0 = l/5 = 20.0$: $\omega_{rk,\max} = -1.848\ m$ when $\gamma = 1$; and $\omega_{lr,\max} = -5.000\ m$ when $\gamma = 10$.

A comparative analysis of formulae (23) and (24) shows the maximal displacements of the right cable part free of loading *p* to be larger than analogous ones of the cable loaded part, i.e. $|\omega_{rk,\max}| > |\omega_{lk,\max}|$. This, probably unexpected result can be explained by always negative middle span displacement Δf_k. This was briefly discussed in [14].

Compare the above displacements developed for the above described cable. The relative difference in percentage of $\omega_{lk,\max}$ vs $\omega_{rk,\max}$ is presented via the graph of Fig 6 for cable of $l = 100\ m$ and $f_0 = 10\ m$: One can find that the relative error gradually increases with the increment of γ: in case of $\gamma = 1$ the $\omega_{rk,\max}$ is larger $\omega_{lk,\max}$ by 28 %; in case of $\gamma = 5$ the $\omega_{rk,\max}$ is larger $\omega_{lk,\max}$ by 70 %; in case of $\gamma = 10$ the $\omega_{rk,\max}$ is larger $\omega_{lk,\max}$ by 86 %.

One must remind the reader that vertical kinematic displacement magnitudes are obtained to be equal ones in absolute values for cable left and right parts if they are determined via engineering methods ([13–18, 21]). Analyze a relative error produced by valuating maximal vertical kinematic displacements (obtained by engineering methods) in respect of cable primary sag f_0 and loads ratio γ. Numerical simulations resulted that no influence of f_0 on a relative error was observed. The magnitude of γ had an essential contribution to relative error magnitude (see graphs of Fig 7 for cable $l = 100\ m$ and $f_0 = 10\ m$). Valuating $\omega_{lk,\max}$: for $\gamma = 1$ the error is 15.5%; for $\gamma = 5$ the error is 42%; for $\gamma = 10$ the error is 52%. Valuating $\omega_{rk,\max}$: for $\gamma = 1$ the error is 10%; for $\gamma = 5$ the error is 16%; for $\gamma = 10$ the error is 17%.

The analysis of the above graphs concludes that admitted tolerance for error obtained employing engineering methods to estimate vertical displacements, can be reached only in case of $\gamma < 1$.

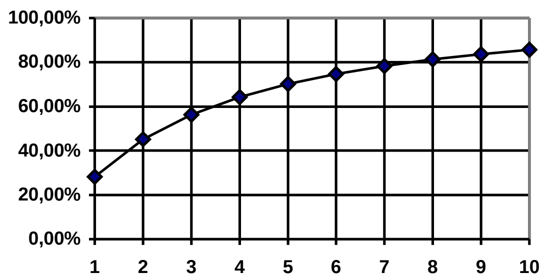

Fig 6. Cable ($l = 100\ m$, $f_0 = 10\ m$) relative difference $|\omega_{lk,\max} - \omega_{rk,\max}|$ in % vs γ

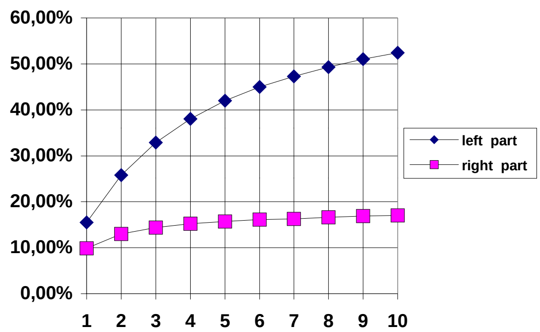

Fig 7. Cable ($l = 100\ m$, $f_0 = 10\ m$) relative error in % of vertical maximal displacements (obtained via engineering methods) vs γ

4. Cable elastic and total displacements

The main cable design task is to design a structure satisfying stiffness conditions per all available loadings during the structure maintenance period ([1, 3, 8, 9, 19]), as it was fixed in introduction. To obtain relatively simple and sufficiently accurate expressions available for practical design purposes certain approximations in cable calculation are to be introduced. For this purpose the principle of superposition of the specific sequence is employed. Firstly, the cable shape (kinematic displacements) due asymmetric loading is determined; secondly, the elastic displacements are determined due to this new – adapted to lading cable shape. The total displacements then are obtained as the sum of kinematic and elastic ones.

The elastic displacements are caused by thrusting force H_1, as it was mentioned above. The compatibility equation of strains yields:

$$\Delta s_g + \Delta s_{el} = 0, \tag{25}$$

here

$$\Delta s_g = s_1 - s_{k1} \tag{26}$$

and

$$\Delta s_{el} = \frac{H_1\, s_0}{EA}, \tag{27}$$

where s_1 and s_{k1} are cable lengths prior and after its deformation, respectively.

Cable length prior deformation is defined by (8); that of after deformation by:

$$s_1 \cong l + \frac{8}{3}\frac{f_1^2}{l}\left[\frac{1+\gamma+5\gamma^2/16}{1+\gamma+\gamma^2/4}\right]. \tag{28}$$

The cable total sag at middle span f_1 can be expressed as the sum of kinematic f_{k1} and that of elastic Δf_{el} ones:

$$f_1 = f_{k1} + \Delta f_{el}. \tag{29}$$

Kinematic displacement f_{k1} is defined by (9) or (4). Then the total sag at a middle span is obtained by:

$$\Delta f = \Delta f_k + \Delta f_{el}. \tag{30}$$

When combining (25)–(29), one obtains an expression to determine the elastic sag at a cable middle span:

$$\Delta f_{el}^2 + 2 f_{k1}\Delta f_{el} - \frac{H_1\, s_{k0}}{EA}\,\frac{3l\left(1+\gamma+\gamma^2/4\right)}{8\left(1+\gamma+5\gamma^2/16\right)} = 0, \tag{31}$$

where

$$H_1 = \frac{ql^2(1+\gamma/2)}{8(f_{k1} + \Delta f_{el})}. \tag{32}$$

One can note that the solution of (31) for Δf_{el} yields the known third order (cubic) equation ([3, 8, 10, 14]). Δf_{el} also can be obtained per iterative procedures. But one must note that both solution ways require significant computational resources, therefore they are not always acceptable in cable design.

Applying the equation (31) and combining (32) one can obtain a simplified (approximate) formula for middle point elastic sag determination:

$$\Delta f_{el} \approx \frac{3}{128} \frac{ql^4 (1+\gamma/2)^3}{EA\, f_k^2 \left(1+\gamma+5\gamma^2/16\right)}. \qquad (33)$$

Find that (33) is analogous to the known approximate formula for estimating the elastic middle point sag in the case of symmetric loading.

The formula (33) enables direct (without supplement calculations) determination of elastic displacement in the case of known kinematic sag f_{k1}. The expression (33) can also serve choosing the cable cross-sectional area, aiming to fit the cable to stiffness requirements. The formula is universal in respect of its application, i.e. for symmetric and asymmetric loadings. In the case of symmetric load ($\gamma = 0$) formula (33) becomes the known equation ([8,10]).

The analysis of formulae (30) and (33) concludes that total displacement at middle span Δf can obtain positive and negative magnitudes due to load intensities magnitudes, their ratio γ and cable axial stiffness *EA*. One can choose the magnitudes of above parameters to obtain $\Delta f = 0$, resulting in equal (in absolute values) vertical displacements for both cable parts. It is evident that purposeful handling of cable primary sag f_0, its stiffness *EA* and loads ratio γ results in the desirable shape and stress state of cable. One can obtain the set of above parameters resulting the cable left part vertical displacements to be larger (in absolute) values vs the ones of the right part.

5. Concluding remarks

A geometrically nonlinear analysis of a cable is of specific nature. It is conditioned by cable ability to response loading only via shape, able to resist tension. Total displacements consist of kinematic (cable shape adapting to loading) and elastic (result of axial deformations) counterparts. To identify the total displacements the following sequence is proposed: first, identification of kinematic displacements; second, determination of elastic displacements according to axial forces compatible the cable shape already adapted the loading. The total displacements are the sum of kinematic and elastic ones. Such an approach of determining actual displacements ensures sufficient accuracy and compatibility with actual practical design of cables.

The developed analytical expressions are proposed to identify kinematic displacements for both parts of an asymmetrically loaded cable, including maximal ones and their location points. It is proved that maximal (in absolute values) kinematic displacements develop in an unloaded cable part and that the cable middle span displacement is always negative (directed up). The obtained analytical expressions for cable middle span displacement (vertical and horizontal ones), employed in obtained relations, ensure obtaining of any cable point kinematic displacements. Horizontal and vertical kinematic displacements are related. It was defined that cable middle span displacement depends on a cable primary sag, symmetric and asymmetric loads ratio. Thus, cable stabilization can be obtained by reducing the above mentioned parameters.

The approximate solution reducing computational efforts and compatible with engineering practice for elastic displacement evaluation is presented.

The obtained formulae enable: handling of the loaded cable shape and its stress state by varying the primary sag, axial stiffness of cable and the loads ratio; choosing of the cable cross-sectional area to ensure stiffness and strength conditions prescribed by design codes.

The performed numerical simulations proved that widely employed engineering methods ensure admissible error when estimating cable displacements only for cases when $\gamma < 1$.

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APKRAUTO LYNO APYBRAIŽOS NUSTATYMAS PER PILNUOSIUS POSLINKIUS

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S a n t r a u k a

Kabamosios konstrukcijos yra labai efektyvios (ekonominiu atžvilgiu), kai naudojamos dideliems tarpatramiams perdengti. Jos gali būti sudarytos iš atskirų kabamųjų lynų arba jų sistemų. Palyginti su įprastiniais laikančiųjų konstrukcijų elementais, kabamojo lyno patrauklumą atskleidžia mažiausias santykis tarp jo laikomosios galios ir savojo svorio. Specifinė lyno savybė ta, kad jis gali dirbti tik tempimui (jis praktiškai neturi standumo lenkimui). Todėl lynas, veikiamas nepusiausvirosios apkrovos, keičia savo pradinę apybrėžą, kad prisitaikytų prie jos, sukeliančios tik tempimo įrąžą. Šis prisitaikymas lemia didelius netiesinius poslinkius. Taigi lyno geometriškai netiesinės elgsenos pobūdis skiriasi nuo standžių konstrukcijų netiesinės elgsenos. Nors abiejų tipų elementuose pasireiškia nedidelės deformacijos, dideli lyno poslinkiai yra sukeliami adaptacinio formos pasikeitimo, o standžiuosiuose elementuose didelius poslinkius sukelia tik deformacijos. Reikia pabrėžti, kad lyno tampriosios deformacijos paprastai yra mažesnės už analogiškas standžiuosiuose elementuose, bet lyno poslinkiai yra gerokai didesni. Taigi esminis apkrauto lyno elgsenos trūkumas yra dideli poslinkiai, kuriuos lemia asimetrinės apkrovos (būdingos tokioms konstrukcijoms, kaip kabamieji tiltai, stadionų stogų perdangos ir t. t.). Todėl projektuojant kabamąsias konstrukcijas svarbiausios yra standumo sąlygos. Gana tikslaus ir patikimo metodo sukūrimas realių kabamųjų konstrukcijų poslinkiams nustatyti yra neabejotinai aktualus. Nustatant pilnuosius (suminius) poslinkius, sumavimo principas realizuojamas tam tikra seka. Poslinkiai skaidomi į du komponentus: kinematinį ir tamprųjį. Pirmasis atsiranda dėl lyno formos pasikeitimo, jam adaptuojantis prie asimetrinės apkrovos pobūdžio, antrasis – dėl tampriųjų deformacijų. Kiekvienas simetriškai apkrauto lyno poslinkis gali būti išreikštas naudojant vidurinį lyno poslinkį. Pateiktos patobulintos analitinės išraiškos viduriniam lyno poslinkiui nustatyti. Jos leidžia nustatyti didžiausius lyno poslinkius ir jų vietas. Pateiktos išraiškos suminiams poslinkiams nustatyti yra patikrintos skaitiškai, atlikta lyginamoji analizė siekiant įvertinti atskirų parametrų, nusakančių poslinkio didumą, indėlį. Aptartos poslinkių nustatymo paklaidos ir jų priežastys, gaunamos plačiai taikant apytikslius inžinerinius metodus

Raktažodžiai: kabamojo lyno apybraiža, netiesinė analizė, asimetrinė apkrova, pilnieji poslinkiai.

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