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ROUTE PLANNING BASED ON UNCERTAIN INFORMATION IN TRANSPORT NETWORKS

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Abstract. The goal of this paper is to find a solution for route planning in a transport network where the network type can be arbitrary: a network of bus routes, a network of tram rails, a road network or any other type of a transport network. Furthermore, the costs of network elements are uncertain. The concept is based on the Dempster–Shafer theory and Dijkstra's algorithm which helps with finding the best routes. The paper focuses on conventional studies without considering traffic accidents or other exceptional circumstances. The concept is presented by an undirected graph. In order to model conventional real transport, the influencing factors of traffic congestion have been applied in the abstract model using uncertain probabilities described by probability intervals. On the basis of these intervals, the cost intervals of each road can be calculated. Taking into account the uncertain values of costs, an algorithm has been outlined for determining the best routes from one node to all other nodes comparing cost intervals and using decision rules that can be defined by the end user, and if necessary, node by node. The suggested solution can be applied for both one type of network as well as for a combination of a few of those.

Keywords: Dijkstra's algorithm, Dempster–Shafer theory, transportation planning, cost interval, route search.

1. Introduction

A number of works deal with choosing routes (Frejinger, Bierlaire 2007; Burinskienė 2009; Jakimavičius, Burinskienė 2009) and navigation (Dee, Hogg 2009) in the transportation area; however, only a small part of those takes uncertainty into account. Literature defines uncertainty information as a stochastic phenomenon (Lin 2009a, 2009b), a fuzzy value (Ji *et al.* 2007; Su *et al.* 2008) or is given with two limit values (Demetrescu, Italiano 2006).

There are some publications on route planning methods based on multi-criteria or/and uncertain information. One of these works describes a learning-based model of route-choice behaviour when information is provided in real time (Ben-Elia, Shiftan 2010), whereas other studies analyze the behaviour of choosing a route when travel time is uncertain (De Palma, Picard 2005) (Henn, Ottomanelli 2006). Other papers present an interactive method for analyzing the multicriteria shortest path problem; nevertheless, this work does not consider uncertain information (Granat, Guerriero 2003). Also, a study on multicriteria adaptive path problems where arc attributes are stochastic and time-varying is provided (Opasanon, Miller-Hooks 2006). None of those offers the opportunity to choose (during the route finding procedure) for a driver thus providing information about the total range defining the parameters of the best route. The aim of research published in this paper is to solve the routing problem based on uncertainty information derived from a lack of information.

Transportation network routing is a problem occurring in many tasks: ware transportation among the cities by lorries, public transportation within the city, individual driving, etc. The aim of the paper is to find a route of the lowest cost in a road network. The concept of this paper is based on previous work (Szűcs, Sallai 2009) where the road network is represented by a graph given with the ordered pair $G = (V, E)$ comprising set V of vertices or nodes together with set E of edges (roads) connecting two nodes. The task of the article is to reach node C from node A in the graph at the lowest cost. The task can be also defined by a directed and undirected graph; nevertheless, this paper presents only the undirected one. The paper focuses on conventional studies without considering traffic accidents or other exceptional circumstances. The graph and predefined costs as the original costs of edges are given; an example is provided in Fig. 1. The costs of edges are changeable (costs are larger than the original costs) according to a lack of information. The goal of research is to develop a route planning procedure taking into account a shortage of information.

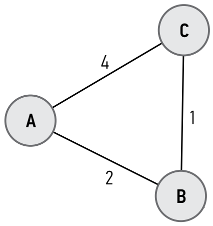

Fig. 1. The example of a diagram showing the predefined costs of edges

Classical Dijkstra's (1959) and improved Dijkstra's algorithm (Xu *et al.* 2007; Cantone, Faro 2004) would give the shortest path in the graph with non-negative edge path costs; however the model described in this paper also contains uncertainty about cost. Therefore, the Dempster–Shafer (DS) theory (Dempster 1968, 2008; Shafer 1976) of evidence is needed, which deals with uncertainty about belief functions.

2. Dempster–Shafer Theory

The Dempster–Shafer (DS) theory denotes different states of a system by H_i similarly to the classical probability theory. In the set $\Omega=\{H_1, H_2, ..., H_n\}$ of all possible states of the system, H_1, …, H_n are still mutually exclusive. Power set 2^Ω is denoted by $P(\Omega)$ and element $P(\Omega)$ by A; thus, A is a composite event:

$$P(\Omega)=2^\Omega=\{\{\},\{H_1\},\{H_2\},\{H_3\},...,\{H_1,H_2\},...,\Omega\}. \quad (1)$$

The DS theory defines functions m (also referred to as *mass*) called basic belief assignment (BBA) on $P(\Omega)$ for expressing the proportion of all relevant and available evidence that supports the claim that the actual state belongs to A but not to the particular subset of A:

$$m: 2^\Omega \to [0,1]; \quad (2)$$

$$A \to m(A). \quad (3)$$

Therefore, it enables to work with non-mutually exclusive pieces of evidence represented by power set $P(\Omega)$. The basic belief assignment (m) function has to satisfy:

$$m(\phi)=0; \quad (4)$$

$$\sum_{A\in P(\Omega)} m(A)=1. \quad (5)$$

Sets A where $m(A) \neq 0$ are called focal elements. $m(A)$ can be interpreted as the degree of the belief given to A and to none of its subsets. In other words, $m(A)$ represents the proportion of evidence that the actual state belongs to A but there is no knowledge about the evidence of the subsets of A. Using the DS theory, a lower and upper limit on $prob(A)$ and the real probability of evidence can be defined.

Belief function $Bel(A)$ of set A is defined as the sum of all the BBA (basic belief assignment) of the subsets of A taking into account that a portion of the belief assigned to B (B is a subset of A) must be assigned to other hypothesis it implies:

$$Bel(A)=\sum_{B|B\subseteq A} m(B). \quad (6)$$

The DS theory also defines plausibility $Pl(A)$ as the sum of all the BBA of sets B (where B is an element of power set) that intersects the set of A:

$$Pl(A)=\sum_{B|B\cap A\neq 0} m(B). \quad (7)$$

Then, the measures are related to each other as follows, where $prob(A)$ is the probability of A:

$$Bel(A)\le prob(A)\le Pl(A). \quad (8)$$

The relationship between the plausibility of set A and the belief of the complement of set A is derived as:

$$Pl(A)=1-Bel(\overline{A}). \quad (9)$$

3. New Model for Route Planning with Uncertain Information

Road traffic is the most important information for routing in the transportation area. This can be simplified considering two states whether it is congestion or not. Thus, the model presents two states (hypotheses) investigated in the DS theory: *Congestion* (*CO*) and *No Congestion* (*NC*), i.e.: $\Omega=\{CO, NC\}$, $P(\Omega)=\{\{\},\{CO\},\{NC\},\{CO, NC\}\equiv\Omega\}$ can be characterised by the BBA values of focal elements $m(CO)$, $m(NC)$ and $m(\Omega)$ where $m(\Omega)$ expresses uncertainty.

In case of congestion on a given road, cost is considered *n-times* of the predefined cost (cost represents transfer time on the road as seen in Fig. 1), otherwise cost remains the predefined cost. (e.g. if n is 2, then, the cost of congestion is twice of the predefined so called original cost.)

The evolving traffic jam (congestion) in the investigated case can be caused by the following different influencing facts (factors):

- weather;
- vehicle density;
- closed lane.

Regarding the simplified situation, the above factors are binary variables: the values of the *weather* are bad or good, the values of *vehicle density* are nominal or high and the values of *a closed lane* are *yes* or *no*. The basic belief assignment (BBA) functions are as follows:

- m_1: bad weather;
- m_2: high vehicle density;
- m_3: closed lane.

BBA functions are given taking into account each edge. An example of all values in view of the graph in Fig. 1 is shown in Table 1.

The values presented in the table and suggesting the DS theory mean that the investigated hypothesis is true with how much probability, e.g. $m_3(CO)=0.7$ means that the fact of *a closed lane* supports the hypothesis of congestion with the probability of 0.7 and $m_3(\Omega)=0.2$ is the uncertainty of evidence. These values are estimated with reference to the previously gathered statistical data using relative frequency calculation.

Table 1. BBA functions for 3 edges

m	Edges		
	<u>AC</u>	<u>AB</u>	<u>BC</u>
$m_1(CO)$	0.40	0.50	0.30
$m_1(NC)$	0.20	0.20	0.20
$m_1(\Omega)$	0.40	0.30	0.50
$m_2(CO)$	0.80	0.60	0.90
$m_2(NC)$	0.05	0.10	0.05
$m_2(\Omega)$	0.15	0.30	0.05
$m_3(CO)$	0.70	0.75	0.60
$m_3(NC)$	0.10	0.05	0.15
$m_3(\Omega)$	0.20	0.20	0.25

If more than one factor appears on the edge, they can be cumulated based on the following formula where A is the investigated set, B, C are the elements of $P(\Omega)$ and m_i, and m_j are the basic belief assignment functions:

$$m_{i,j}(A)=\frac{\sum_{B\cap C=A} m_i(B)\cdot m_j(C)}{1-\sum_{B\cap C=0} m_i(B)\cdot m_j(C)}. \quad (10)$$

The cost of the edge will be the sum of the predefined fixed cost (represents transfer time on an empty road) and uncertainty cost from DS calculation. The DS theory gives a DS interval for each set: *Bel* and *Pl*, the lower and upper limits (DS limits) of the likelihood of evidence for the concerned set. Uncertainty cost will be calculated by the product of the fixed cost and DS limits.

4. Calculations

Let us assume that only a *closed lane* is true for edge <u>AC</u>, *high vehicle density* and a *closed lane* are true for edge <u>AB</u>, and *high vehicle density* is true for edge <u>BC.</u> In this session, the calculations of the costs of edges using the values given in Table 1 are shown. The ratio of the maximal and predefined cost of each edge is also *n*.

A. Investigation into edge <u>AC</u>:

$$Bel(CO)=0.7;$$
$$Pl(CO)=1-Bel(NC)=1-0.1=0.9;$$
$$Cost(\underline{AC})=e(\underline{AC})\times$$
$$\left[1+(n-1)\cdot Bel(CO);\ 1+(n-1)\cdot Pl(CO)\right]=$$
$$4\cdot\left[0.3+n\cdot 0.7;\ 0.1+n\cdot 0.9\right];$$

B. Investigation into edge <u>AB</u>:

The cumulative formula should be used considering two factors:

$$m_{2,3}(A)=\frac{\sum_{B\cap C=A} m_2(B)\cdot m_3(C)}{1-\sum_{B\cap C=0} m_2((B)\cdot m_3(C)}. \quad (11)$$

Sets A, B and C are the subsets of all possible states of the system. The products of the BBA values of the subsets are shown in Table 2.

Table 2. BBA Functions having 2 factors

$m_2\cdot m_3$		m_3		
		CO	NC	Ω
m_2	CO	0.450	0.030	0.120
	NC	0.075	0.005	0.020
	W	0.225	0.015	0.060

The intersection of the following pairs will be empty: $(B=\{CO\}, C=\{NC\})$ and $(B=\{NC\}, C=\{CO\})$. Thus, the denominator will be: $1-(0.03+0.075)=0.895$. The nominator is:

$$\sum_{B\cap C=A} m_2(B)\cdot m_3(C); \quad (12)$$

if set A is CO: $0.45+0.12+0.225=0.795$;

if set A is NC: $0.005+0.02+0.015=0.04$;

if set A is Ω: 0.06.

Using the nominators and denominator the results are:

$$m_{2,3}(CO)=0.888;$$
$$m_{2,3}(NC)=0.045;$$
$$m_{2,3}(\Omega)=0.067;$$
$$Bel(CO)=0.888;$$
$$Pl(CO)=1-0.045=0.955;$$
$$Cost(\underline{AB})=e(\underline{AB})\times$$
$$\left[1+(n-1)\cdot Bel(CO);\ 1+(n-1)\cdot Pl(CO)\right]=$$
$$2\cdot\left[0.112+n\cdot 0.888;\ 0.045+n\cdot 0.955\right].$$

C. Investigation into edge <u>BC</u>:

$$Bel(CO)=0.9;$$
$$Pl(CO)=1-0.05=0.95;$$
$$Cost(\underline{BC})=e(\underline{BC})\times$$
$$\left[1+(n-1)\cdot Bel(CO);\ 1+(n-1)\cdot Pl(CO)\right]=$$
$$1\cdot\left[0.1+n\cdot 0.9;\ 0.05+n\cdot 0.95\right].$$

The cost of the route of edges <u>AB</u>, <u>BC</u> is the sum of the interval of serial edges:

$$Cost(\underline{AB},\underline{BC})=$$
$$2\cdot\left[0.112+n\cdot 0.888;\ 0.045+n\cdot 0.955\right]+$$
$$1\cdot\left[0.1+n\cdot 0.9;\ 0.05+n\cdot 0.95\right]=$$
$$\left[0.324+n\cdot 2.676;\ 0.14+n\cdot 2.86\right].$$

A comparison of the above with the interval of another cost:

$$Cost(\underline{AC})=\left[1.2+n\cdot 2.8;\ 0.4+n\cdot 3.6\right]$$

indicates that the route of edges <u>AB</u>, <u>BC</u> is always better (shorter) for each *n* (*n*>1), because the lowest value of interval <u>AC</u> is worse than the highest cost of the other route. Therefore, the route of the lowest cost presented in this example is the route of edges <u>AB</u>, <u>BC</u>.

5. Route Planning Algorithm with Uncertain Information

Making a decision of the compared interval was easy in the previous example, because the lowest value of one of them was higher than the highest value of the other one. If there is an overlap between the intervals, then the decision is not always easy. If both end points of the interval are lower than the end points of the other interval, the routing procedure could decide for lower values. However, if one interval is an inner part of another interval, the decision is ambiguous. One possible election is a comparison of the middle points of the intervals. Election rules depend on the human decision maker: the end user can develop the worst or the best case design or other types of design for making decisions.

An algorithm with uncertain information has been developed for route planning; a simplified version is presented in the block diagram in Fig. 2. The algorithm calculates the intervals of the lowest cost for each node from the source node where S is the set of the examined nodes, *T* is the set of remaining (not examined) nodes in the graph and *s* is the source (start node). Proc DS procedure calculates cost values with uncertainty based on the DS theory and the outputs of this procedure are cost intervals.

The next stage of the algorithm presents a circle for all *u* nodes where that node will be selected and the cost of which is minimal. Next, all the neighbours of *u* will be used for comparison purposes in order to find the minimal cost of *u*. If the result of the comparison is unambiguous, then, cost intervals will be determined, otherwise an automatic decision process (*Proc_autoDecision*) will calculate the values based on the predefined rules, or the human decision maker will be provided a possibility of overwriting them. At the end of the circle, node *u* will be examined and transferred from set *S* to set *T*. If the set (*T*) of not examined nodes is empty, then, the algorithm terminates. The obtained results will be the best cost intervals for all nodes. The running time of the algorithm is $O(|V|^2 + |E|)$ (which is not very fast); however, a big advantage of the procedure is knowledge about dispersion (deviation) around the expected value of the received result (a user gets an interval instead of one value).

Combination of any type of network

People use vehicles, city trains and public transport in cities, which leads to the problem of multimodal transport. Such complex situation requires an explanation for route planning. In order to deal with multimodal transport, a simple solution has been used: a large diagram was drawn from the standalone graphs. The combined diagram is the union of (i) different types

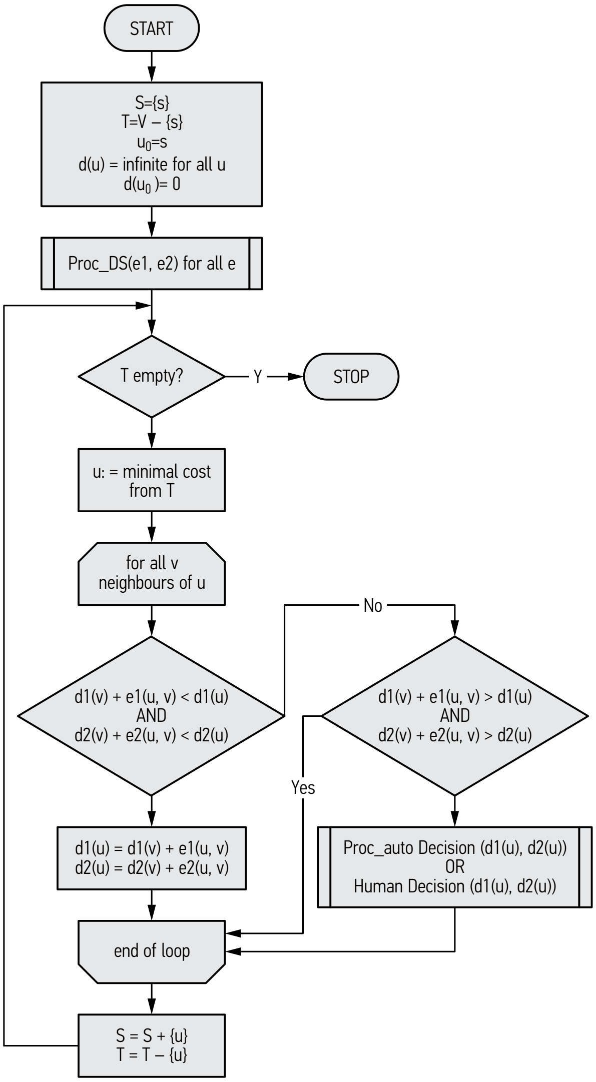

Fig. 2. Route planning algorithm based on uncertain information

of the network as diagrams and (ii) connections among them. These *connection* edges represent transfer possibilities from a type of transport mode to another. Let us see a little example for the diagram combination.

Fig. 3 shows four types of the network (*a* – road, *b* – tram, *c* – railway, *d* – metro) and some connection possibilities among them: node 1 and 13, node 8 and 13, node 9 and 14, node 12 and 13, node 12 and 16, node 16 and 20. Furthermore, there are some similar nodes in different networks, e.g. transport mode can be changed at once (without any additional cost) in node 9. Fig. 4 represents the combined diagram where the union of small graphs is indicated.

Example

The algorithm described above has been tested in small and medium-sized diagrams. Fig. 4 shows the investigated combined network, including costs (original cost without taking uncertain information into account) built from four diagrams displayed in the previous example by combining them.

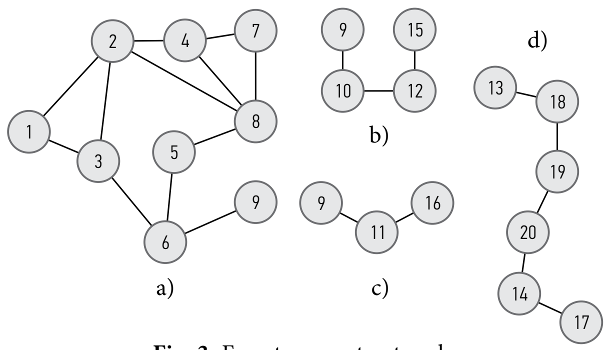

Fig. 3. Four transport networks

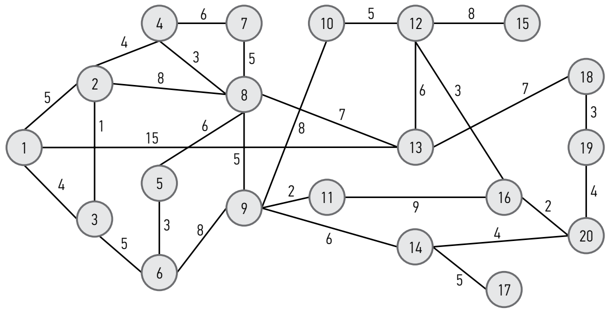

Fig. 4. Road network including costs

Let us assume that the influencing factors and their uncertainties are given as seen in Table 3. The first column contains edges while the second one shows actual facts for a given edge where I means bad weather, II – high vehicle density, III – a closed lane. The last 9 columns contain BBA values (m).

The implemented algorithm described in the previous section calculates cost intervals. Fig. 5 shows the network with the calculated cost intervals of each edge (in case of $n = 2$) based on BBA and facts. Small digits correspond to edges, whereas the large ones show the best cost intervals for all nodes, for example, the best route from nodes 1 to 20 is 1–2–8–9–11–16–20, and the best cost interval is [36.56, 40.14].

Table 3. BBA functions and real facts

		m_1			m_2			m_3		
edges	facts	*CO*	*NC*	Ω	*CO*	*NC*	Ω	*CO*	*NC*	Ω
1–2		0.50	0.20	0.30	0.70	0.10	0.20	0.65	0.10	0.25
1–3	III	0.60	0.10	0.30	0.65	0.20	0.15	0.80	0.05	0.15
1–13	I; II	0.65	0.20	0.15	0.50	0.10	0.40	0.75	0.10	0.15
2–3	II; III	0.80	0.05	0.15	0.75	0.20	0.05	0.60	0.20	0.20
2–4	III	0.70	0.10	0.20	0.85	0.10	0.05	0.65	0.20	0.15
2–8		0.55	0.05	0.40	0.70	0.10	0.20	0.60	0.10	0.30
3–6	I	0.45	0.20	0.35	0.80	0.05	0.15	0.85	0.10	0.05
4–7	I; II; III	0.60	0.15	0.25	0.60	0.10	0.30	0.90	0.05	0.05
4–8	II	0.40	0.20	0.40	0.90	0.05	0.05	0.70	0.10	0.20
5–6	III	0.50	0.10	0.40	0.80	0.10	0.10	0.90	0.05	0.05
5–8	I; III	0.65	0.05	0.30	0.70	0.10	0.2	0.65	0.20	0.15
6–9		0.75	0.10	0.15	0.65	0.10	0.25	0.85	0.10	0.05
7–8	I; III	0.45	0.10	0.45	0.75	0.20	0.05	0.60	0.10	0.30
8–9	III	0.50	0.30	0.20	0.85	0.10	0.05	0.90	0.05	0.05
8–13	II	0.60	0.15	0.25	0.90	0.05	0.05	0.80	0.10	0.10
9–10	II	0.65	0.05	0.30	0.80	0.05	0.15	0.70	0.10	0.20
9–11	I; II; III	0.40	0.10	0.50	0.70	0.20	0.10	0.80	0.05	0.15
9–14	III	0.55	0.10	0.35	0.60	0.20	0.20	0.75	0.10	0.15
10–12	III	0.60	0.10	0.30	0.70	0.10	0.20	0.80	0.15	0.05
11–16		0.50	0.10	0.40	0.75	0.10	0.15	0.60	0.10	0.30
12–13	II	0.45	0.25	0.20	0.80	0.05	0.15	0.75	0.15	0.10
12–15	I	0.60	0.15	0.25	0.85	0.05	0.10	0.90	0.05	0.05
12–16	I	0.35	0.30	0.35	0.65	0.10	0.25	0.80	0.10	0.10
13–18		0.70	0.20	0.10	0.55	0.20	0.25	0.70	0.15	0.15
14–17	II	0.65	0.15	0.20	0.80	0.10	0.10	0.65	0.20	0.15
14–20	I; II	0.40	0.30	0.30	0.70	0.10	0.20	0.85	0.10	0.05
16–20	III	0.75	0.10	0.15	0.65	0.10	0.25	0.60	0.20	0.20
18–19	III	0.45	0.30	0.25	0.80	0.10	0.10	0.70	0.10	0.20
19–20		0.60	0.30	0.10	0.90	0.05	0.05	0.80	0.15	0.05

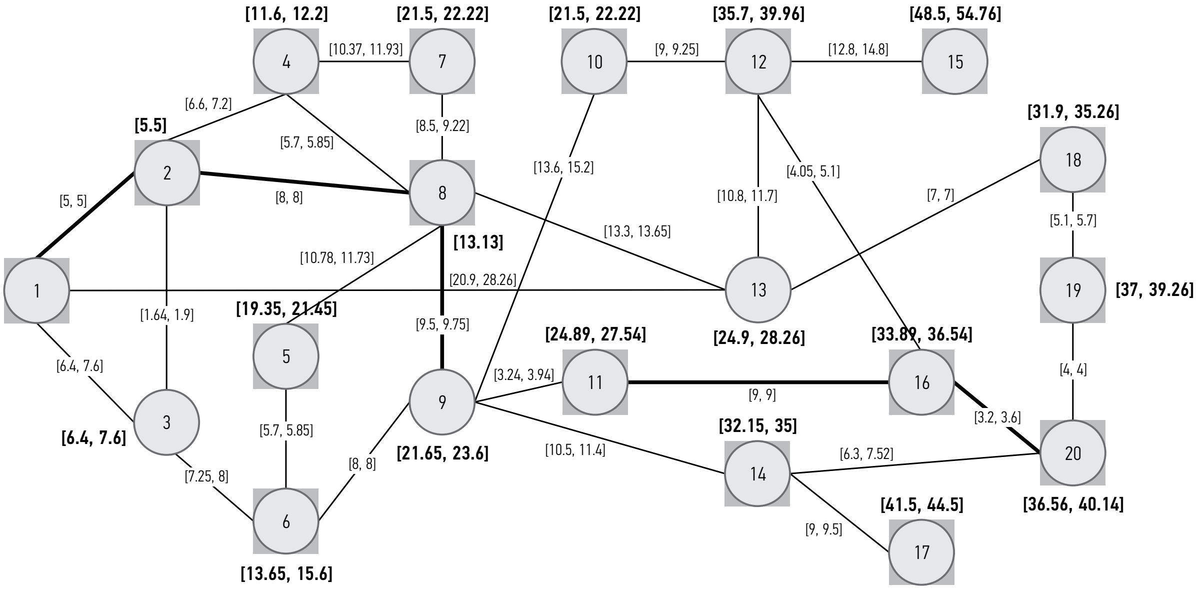

Fig. 5. Results of the algorithm

6. Conclusions

The paper provides a model and an algorithm for routing in any type of the transport network taking into account uncertainty about information on the condition of roads and cost-influencing factors. The improved model is capable of handling multimodal transport as well as of aggregating unimodal transport possibilities. Probability values are uncertain in the model, whereas an algorithm determines and uses probability intervals defined by minimum (belief) and maximum values (plausibility) using the DS theory. While applying these extreme values, cost uncertainty intervals for each road can be calculated and further used for any route.

Besides uncertain information values (written in table format), another parameter (*n*) can be given for the improved algorithm that determines the ratio of maximum and minimum costs. The proposed algorithm presents the best routes from one node to all other nodes comparing cost intervals and using decision rules. Not only the best route but also the cost interval of the best choice can be obtained.

The results of investigation presented in this paper can be used for binary decisions in transport (McCammon, Hägeli 2007), or route planning (Keshkamat *et al.* 2009), where the route planning algorithm takes into account environmental and socio-economic considerations for selecting alternative routes.

The research results can be used for transportation enterprises, public transport companies and individual drivers. The concept can be investigated considering an economical aspect. The input of the concept is estimated data (with probability values) arising from statistical data on traffic under different circumstances (situation types). Producing such input data is expensive because of a large amount of information necessary for each situation. However, if using the concept formulated in this paper, income received by transportation and public transport companies could be larger than the cost of producing these input data. The base of higher income is more sophisticated route planning and a high amount of vehicles; this profit can be realized later.

The research paper is aimed at solving the routing problem based on uncertainty information derived from a lack of data. The novelty of the paper is offering a driver the opportunity for finding the best route and providing detailed information about the total range of the parameter of the best route (information on the cost interval).

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