



BUS STOP LOCATION UNDER DIFFERENT LEVELS OF NETWORK CONGESTION AND ELASTIC DEMAND

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Abstract. The article analyses optimal bus stop locations under different network congestion levels applying a bi-level optimisation model, covering an upper level minimizing an overall cost function (Social Cost) and a lower level that includes a modal split assignment model. This model is applied to Santander city (Spain) under a range of demand levels, starting from very low to high congestion, representing the evolution of variables in each case and analysing different solutions. The optimal distances between stops obtained for each demand and congestion level indicate that very low demands produce wider spaces. However, as demand increases, accessibility to public transport service should be increased and then spacing between bus stops drops to 360 metres.

Keywords: bus stop spacing, bi-level optimization, elastic demand, congestion, transit system optimization.

1. Introduction

Traditionally, the physical and operational design of an urban transit system has been looked at from the routing point of view, frequencies and necessary or available fleet size. However, the location of bus stops, especially in Europe, has been based on experience subjected to space constraints or minimizing the interaction with private traffic.

Evidently, the correct management of all these factors is essential in any efficient usage of available resources, however, it is equally true that when introducing a new public transport system into urban space or modifying the existing one, the location of bus stops, and therefore, walking time, takes on special relevance (Vedagiri, Arasan 2009).

A rational distribution of bus stops would be advantageous not only for public transport service but probably also for the flow of traffic in general.

Several research works in this field, especially in the '70s and '80s (Vuchic, Newell 1968; Kraft, Boardman 1972; Gleason 1975; Lesley 1976; Wirasinghe, Ghoneim 1981; Ceder, Wilson 1986; Ghoneim, Wirasinghe 1987), and various methodologies continued to evolve until the last decade.

For example, Van Nes (2001), Van Nes and Bovy (2000) concentrated their research on fundamental variables (spacing between stops and bus routes) in

designing a public transport network and on how objectives influenced the resulting design values, taking into account the preferences of the traveller and budget restraints. They considered three points of view (passengers, operators and local authorities) to obtain several objective functions that could be used for fixed demand design concluding that the minimization of the overall costs was the most realistic objective when taking on a problem of public transport design.

In the same year, Furth and Rahbee (2000) modelled the impact of modifying spacing between bus stops on a bus line. They used a geographical model to distribute demand and a dynamic programming algorithm to find optimal spacing between the corresponding stops which were then applied to the line in the city of Boston (USA), increasing spacing between stops from 200 to 400 metres.

Saka (2001) proposed a model for determining optimal bus stop spacing using a detailed decomposition of bus journey times taking into account the time spent waiting at stops, acceleration, deceleration etc. and later performed sensitivity analysis of variation between bus stop spacing and its influence on frequency, thereby, arriving at optimal spacing.

Sankar *et al.* (2003) applied multi-criteria analysis using a GIS to consider possible bus stop locations with various parameters like distance, location importance,

user willingness to walk and junctions applied to a route in an Indian city.

Chien and Qin (2004) developed a model applied to a public transport route with demand not distributed uniformly along the entire route and more realistic than previous work, in which the number of stops was calculated with the minimized overall cost and performed sensitivity analysis with respect to various parameters (users' value of time, speed of accessing service and demand density).

Schöbel (2005) sees the problem of bus stop location as a function that tries to maximise coverage with the least possible number of stops through a set-covering problem.

Dell'Olivo et al. (2006) proposes a combined optimization model for frequencies and bus stop spacing which, for the first time, considers the importance of bus capacity constraints, using a fixed and known public transport trip matrix.

The same year, Alterkawi (2006) developed a computer program applied to Riyadh city (Saudi Arabia) which provided optimal bus stop spacing according to passenger access distances and 'bus weaving' in order to reduce many public transport accidents occurring in that city due to a high number of bus stops. Bus stop interaction with traffic was analyzed from a microscopic point of view in various research projects (Koshy, Arasan 2005; Zhao et al. 2007, 2008).

Furth et al. (2007) applied parcel-level modelling to analyze how the relocation of stops along public transport routes impacted access times. Ziari et al. (2007) introduced simplifications in the method to determine bus stop spacing traditionally used in England and suggested a way to reduce spacing between stops without increasing vehicle travel time.

More recent research has appeared following methodologies evaluating public transport systems where, even if bus stops are considered fixed, their location plays an important role in determining the quality of service (Chen et al. 2009; Shimamoto et al. 2010; Hu et al. 2010; Niewczas et al. 2008; Jakimavičius, Burinskienė 2010; Daunoras et al. 2008; Mesarec, Lep 2009; Matis 2010).

Finally, Ibeas et al. (2010) complement Dell'Olivo's work on a bus stop location model combined with a modal distribution model and analyses changes in travelling habits under different bus stop configurations on the network.

The proposed model is based on one made by Ibeas et al. (2010) performing sensitivity analysis by applying those to different scenarios depending on the level of demand and network congestion. This model also complements preceding research that may be applied to the entire, real public transport network under any demand structure; it also considers elastic demand for evaluating variations in modal split as a function of the final bus stop location and models congestion on the public transport system, thereby increasing its applicability.

2. Proposed Model

A bi-level optimization model is proposed to solve the problem of macroscopic bus stop locations. The upper level of the model minimizes a cost function (Z) made up of user costs (UC) and operator costs (OC); a lower level containing a modal split assignment model takes into account the influence of private traffic and congestion on the movement of public transport vehicles (Ibeas et al. 2010).

Upper Level

User costs are made up of respective times: access (TAT), waiting (TWT), travelling, ($TIVT$ by bus and $TCTT$ by car) and transfer (TTT) weighted with their respective coefficients (ϕ). Operator costs are made up of direct costs (rolling costs (CK), personnel costs (CP), hourly costs due to standing still with the engine running (CR), fixed costs (CF)) and indirect costs (exploitation, human resources, administrative-financial, depot and supplies, management and general costs) considered to be 12% (Ibeas et al. 2006) of direct cost.

Rolling costs will be equal to the total number of kilometres covered and multiplied by its unit cost per kilometre.

The cost of stationary buses with the engine running will depend on the time they spend at the bus stop dealing with passengers and traffic signals multiplied by its unit cost per hour.

The total financial fixed costs and personnel costs can be represented as a function of the total number of buses actually circulating and multiplied by their respective unit cost per hour and bus.

The cost structure presented above defines the upper level optimization problem (Ibeas et al. 2010) consisting of minimizing costs proposed so far (1) subjected to any operational constraint.

It seems evident that as the number of bus stops increases, then user access time to public transport decreases and operating costs increase because the time taken to turn a bus round increases requiring a larger fleet or a change in frequencies. This is why these operational constraints may include a maximum operating budget or maximum fleet size in the group of constraints (2).

$$\min Z = \phi_a TAT + \phi_w TWT + \phi_v TIVT + \phi_t TTT + \phi'_v TCTT + 1.12 \cdot (CK + CR + CP + CF) \quad (1)$$

s.t.:

$$C_o \leq C_o^{\max} \\ \sum_l \text{round}^+(tc_l/h_l) \leq fls_{\max}. \quad (2)$$

Lower Level

A lower level is modelled using a combined Mode Choice – Assignment Model (De Cea et al. 2003) considering deterministic user equilibrium (DUE) based on Wardrop's first principle of choosing routes on different modal networks of a public and private transport and a logit type model (multinomial or hierarchical) for making decisions relating to transport mode (De Cea

et al. 2003). Equilibrium flow conditions for this problem considering the application case of two transportation modes (auto and bus) can be formulated using an equivalent optimization problem of the following type (De Cea et al. 2003):

$$\min \sum_a \int_0^{f_a} c_a(x) dx + \sum_s \int_0^{V_s} c_s(x) dx + \frac{1}{\gamma} \sum_m \sum_w T_w^m (\ln T_w^m - 1) \quad (3)$$

s.t.:

$$\begin{aligned} T_w &= \sum_m T_w^m, \forall w; \\ T_w^m &= \sum_{p \in P_w^m} h_p^m, \forall w, m; \\ f_a &= \sum_{p,w} h_p \cdot \beta_{ap}, \forall a; \\ V_s &= \sum_{p,w} h_p \cdot \beta_{sp}, \forall s; \\ h_p^m, V_s, f_a &\geq 0, \forall p, s, a, m, \end{aligned} \quad (4)$$

where: a – network link; s – route section: virtual links between bus stops within the network on which attractive public transport lines run; f_a – flow on link a ; V_s – passenger flow in route section s ; c_a – the cost function of travelling on link a ; c_s – the cost function of users travelling on public transport on route section s ; m – the mode of transport; T_w – a total number of trips between O-D pair w ; T_w^m – a total number of trips between O-D pair w on mode m ; P_w^m – a group of routes associated with O-D pair w on mode m ; h_p^m – flow on route p in mode m ; γ – a parameter to calibrate in decision tree nest (logit); β_{ap} – the element of the link-route incidence matrix (takes value 1 if link a is a part of the route or way p and 0 in other cases); β_{sp} – the element of the route-route section incidence matrix on public transport (takes value 1 if route section s is a part of the route or way p and 0 in other cases).

The first two terms in (3) correspond to car and bus assignments and the third term corresponds to modal split considering a multinomial logit structure.

The group of constraints (4) represents trip conservation constraints, the relationship between flows on routes and flows on links, and constraints on the non negativity of flows.

3. Solution Algorithm

The algorithm developed to solve the optimization problem is based on that developed by Ibeas et al. (2010). This heuristic algorithm is made up of various stages (Fig. 1) over which the bi-level optimization problem is solved. The stages are described below:

Stage 0. This initial stage could be called a ‘network preparation stage’. The road network must be discretized into almost equally long links, thereby defining all potential candidates for locating a bus stop. The study area is then divided

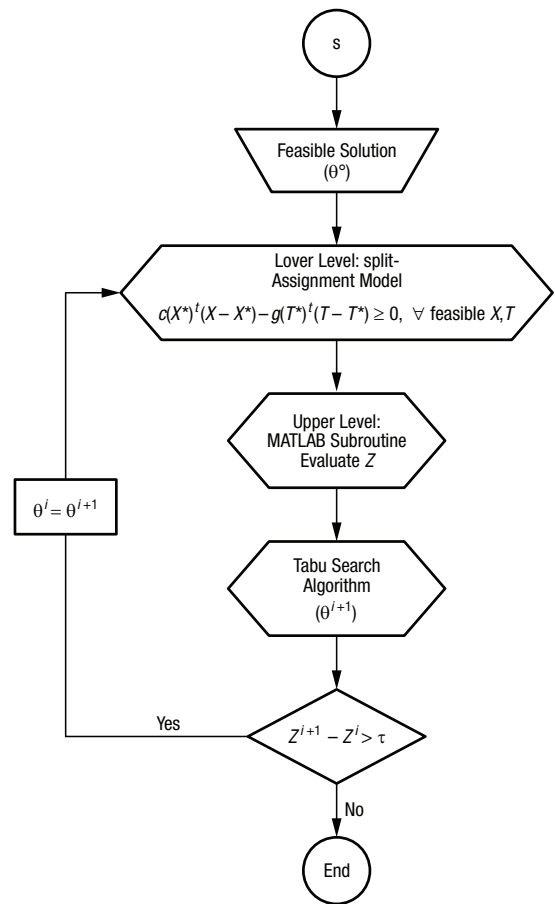


Fig. 1. Solution algorithm

into a number of separate zones each of which must have the same spacing between bus stops. A vector (∂) is thereby created with as many components as defining zones. Details on zoning criteria can be found in Ibeas et al. (2010).

Stage 1. The initial feasible bus stop spacing solution vector (∂^0) is generated at the first iteration.

Stage 2. Given the network configuration of bus stops defined by (∂^i), the lower level optimization problem of the proposed model is solved at each iteration i of the algorithm and the upper level objective cost function is calculated (Z^i).

Stage 3. New values of (∂^{i+1}) are generated using the *Tabu Search* algorithm (Glover 1989) and the upper level problem is solved to determine the new value of Z^{i+1} , from which we return once again to run Stage 2.

Stage 4. If $Z^{i+1} - Z^i > \tau$, we return to Stage 3; however, if $Z^{i+1} - Z^i \leq \tau$, the algorithm is stopped, where τ is a reduction in the value of the objective function established as stop criteria.

Software package ESTRAUSTM (De Cea et al. 2003) is used for the lower level (Mode Choice – Assignment Model) while MATLABTM (www.mathworks.com) is used for calculating the upper level cost function and for running the solution algorithm.

4. Application to a Real Network

To check the validity of the model presented in this article, it has been applied to a real case. The study area used was Santander city, a medium sized city, of approximately 180000 inhabitants located in Northern Spain with a well established urban bus service (Fig. 2). The city is characterised by its linear structure, a well developed urban and commercial centre and various peripheral residential zones with differing population densities.

The network had previously been discretized into 60 m long segments to be as precise as possible when locating each stop, given restrictions on software. A GIS has been developed and included economic, social and demographic characteristics of each city zone as well as its more significant attributes of each node: typology (junction, dummy nodes...), regulation (traffic lights) and location (on a slope or not, the existence of residential areas or special points of interest at a maximum distance if it is an 'obligatory' location for a stop, tunnels, etc.).

The zonal aggregation of the city was based on population density and commercial activity and provided five groups of zones with equal distancing (in metres) between stops.

5. Analysis and Discussion of Results

In order to analyse the influence of demand and congestion in the final solution provided by the model and the relationship between different variables, the starting point was a very low level of demand, which was progressively increased multiplying the O/D matrix of San-

tander by 0.5, 0.75, 1.5 and 2. In this way, we can analyse an interval range of demand varying from half to double of actual demand at peak hour.

Table 1 shows the actual number of trips and their modal split and Table 2 indicates the values of time and the operation unit used (Ibeas *et al.* 2010).

Table 1. Trips per hour by mode in Santander

Mode	Car	Bus	Total
Trips per hour	48 441	4 944	53 385

Table 2. Values of time and operation unit costs

Variable	
Travel time (BUS)	26.43 €/h
Waiting time (BUS)	51.29 €/h
Access time (BUS)	31.01 €/h
Transfer time (BUS)	79.77 €/h
Travel time (Car)	28.90 €/h
CK unit cost	0.4 €/km
CP unit cost	14 €/bus/h
CF unit cost	32 €/bus/h
CR unit cost	0.02 €/h

Various initial spacing vectors were used for each matrix, starting from extreme cases and the initial configuration for Santander to the analyses of how the variables of the model evolved and interacted.

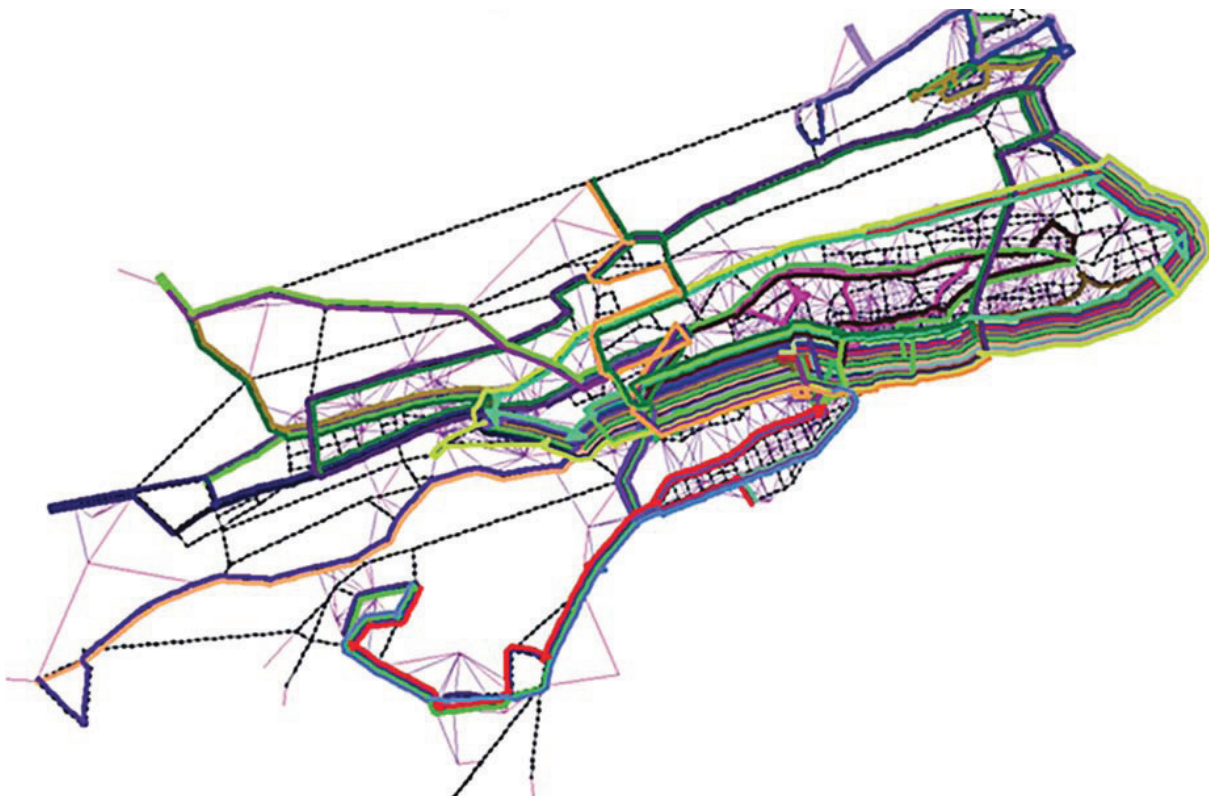


Fig. 2. Road network and public transport routes in Santander

Table 3. The final results of different levels of demand (1)

Matrix	Z	C_{bus}	C_{car}	C_{oper}		Distance (m)					N° stops	Trips bus
						D1	D2	D3	D4	D5		
0.5	165267	39553	123560	2154	1800	1740	1800	1860	1800	68	1580	
0.75	316517	52154	261952	2411	1200	1200	1200	1200	1200	84	2351	
1(Santander)	707318	80805	622659	3854	360	420	540	420	780	153	5109	
1.5	1374611	2859	1366816	4936	360	420	480	420	420	166	10240	
2	2462995	135216	2418825	8954	360	360	360	420	360	184	12281	

Table 4. The final results of different levels of demand (2)

Matrix	C_{bus}/pax	C_{car}/pax	Z/pax	Operator Profit	Fleet
0.5	12.51	2.43	3.06	58.7	35
0.75	13.31	4.12	4.69	331.6	40
1(Santander)	9.95	8.54	8.73	1 825.8	61
1.5	10.27	16.23	14.55	2 232	89
2	11.01	25.28	22.81	-357.3	167

Tables 3, 4 show the final solution given for each level of demand.

These tables show that for very low levels of demand (0.5 and 0.75), bus stops can be uniformly well spaced apart from those throughout the city at approximately every 1200÷1600 metres. These quite high distances should not be taken as rigid measurements between bus stops but rather they should be looked at as the overall bus stop density. In fact, the final result from the model locates bus stops in the zones with higher levels of trip generation /attraction and in the highest populated areas within each zone (more specifically, in places where the greatest number of access links converge). This indicates that the loss of passengers from the bus that moves to using a private car does not cause congestion on the network (because the overall demand is very low); therefore, social cost is compensated using savings made by the operator who now requires a smaller fleet to provide the required service (remember that frequencies are kept constant in the problem). Nevertheless, as demand increases, accessibility to public transport should be greater reducing spacing to 360 metres.

These results may be better understood by representing the evolution of each variable for each configuration of bus stops and each level of demand. The legend used in the graphs is the one shown in Fig. 3.

Fig. 4 shows the evolution of social cost and the number of bus passengers.

Both graphs show that the sensitivity of social cost to the number of stops (or bus stop spacing) increases with higher levels of demand and congestion. The same thing happens with the number of passengers using the public transport system.

In all cases, accessibility is reduced, so does the number of passengers the amount of which increase

along with higher bus stop densities reach a moment when an excessive number of bus stops provokes a reduction in passenger numbers due to an increase in bus journey times. Bus passengers can also be seen to be quite sensitive to longer distances between stops.



Fig. 3. Legend

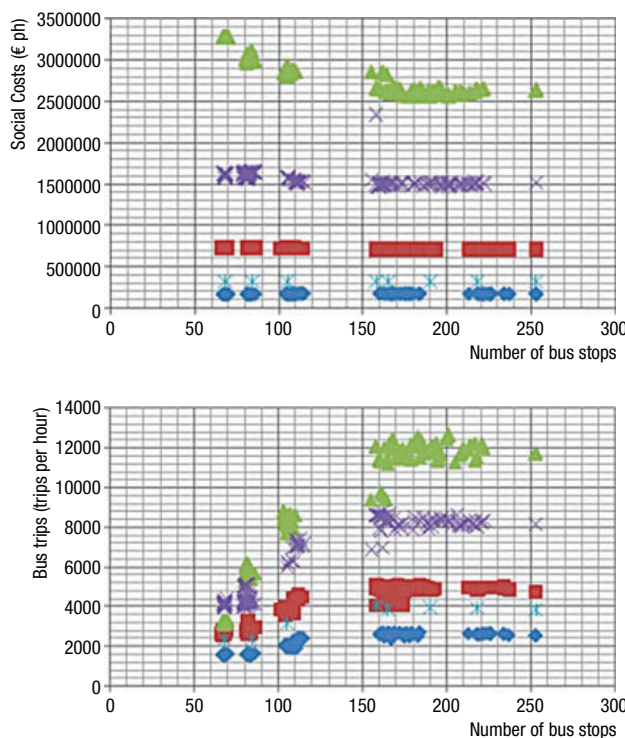


Fig. 4. The evolution of social costs and bus trips

Fig. 5 shows the relationship between the users of public transport and social cost, especially at high demands. This is backed up by the relationship showing that as demand levels increase, the importance of public transport in the overall social cost increases.

In other words, for high demands, any possible gain in the numbers of public transport users due to a reduction in congestion will be beneficial for the network as a whole. As indicated in the right hand graph, car users are seen to benefit by the increased number of passengers because their unit cost is reduced.

The level of demand and spacing between stops also logically influence the public transport operating company as shown in Fig. 6. If the number of stops increases, the operating times of buses also increase; thus, theoretically, if a public transport operating company wish to maintain the given headway, they will have to increase the number of buses.

As the number of stops reduces, the required fleet size is lower. However, depending on the level of de-

mand, the degree of congestion, made worse by the loss of passengers, makes journey times even longer than when there are more stops, making larger fleets necessary. Thus, operating company margins are seen to increase with the number of passengers, but they are also influenced by the fleet size needed to provide service, as shown in Fig. 7.

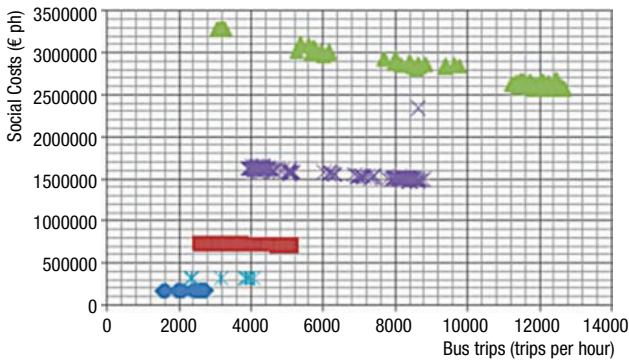


Fig. 5. Relationship between the users of public transport and social cost

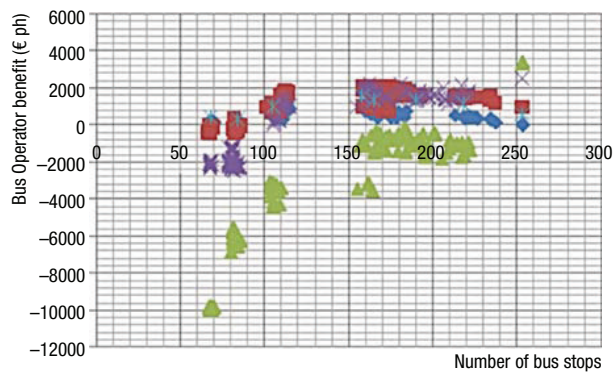
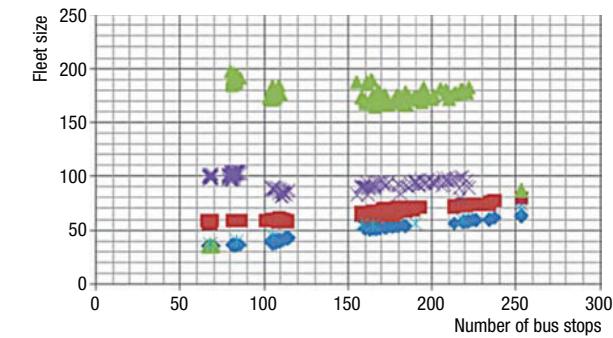


Fig. 6. The evolution of fleet size and bus operator costs

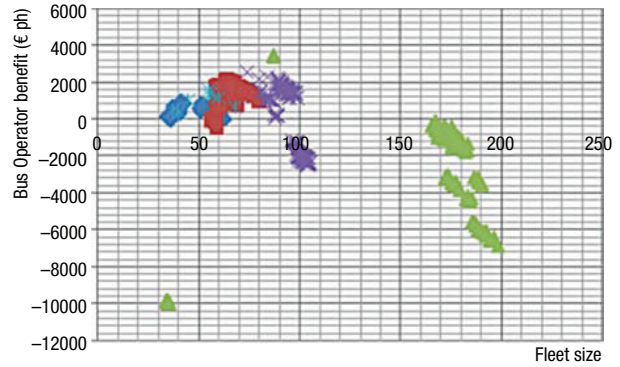


Fig. 7. Relationship between bus operator costs and fleet size

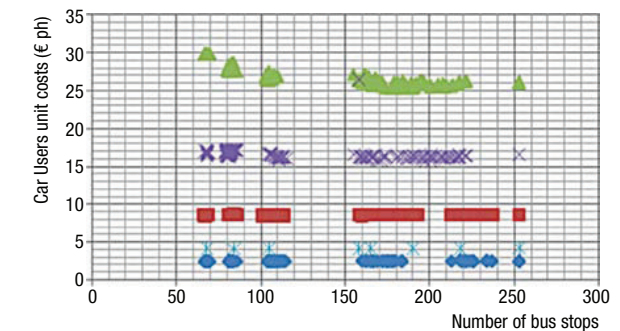
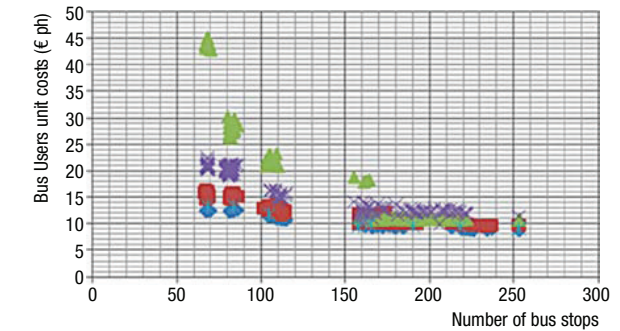
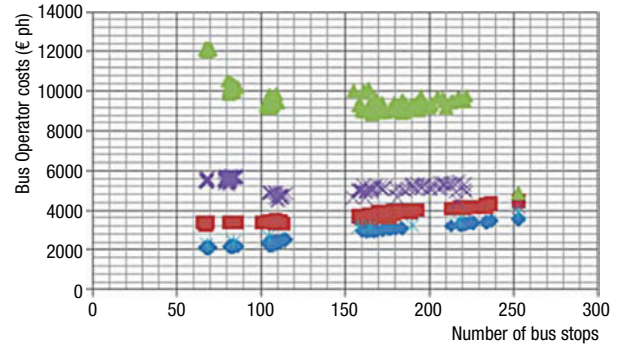


Fig. 8. Relationship between bus operator, bus users, car user costs and the number of bus stops

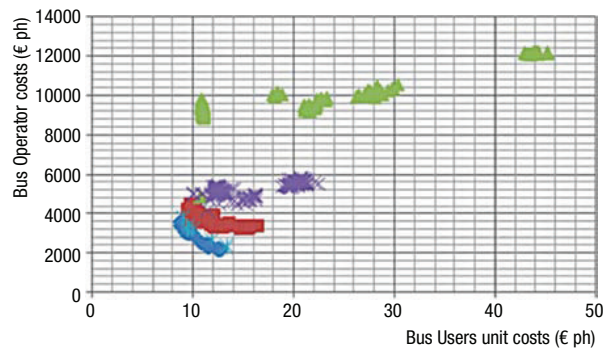


Fig. 9. Relationship between bus operator and bus user costs

Fig. 8 shows some further interesting relationships between variables and how, for low demands, operating cost increases with the number of stops; consequently, the required fleet size increases and as operating costs increase the user costs decrease.

However, as congestion increases, this relationship starts to invert reaching a direct relationship between both costs as shown in Fig. 9. This could give rise to weighting analysis between both costs depending on the degree of congestion in the initial problem.

6. Conclusions

1. This research has analysed the optimal location of bus stops under different degrees of network congestion. The methodology has been applied to a real case (Santander city) and tested under varying demand levels, starting with very low levels of demand and reaching high congestion levels, representing the evolution of different variables in each case and analysing different solutions whilst keeping frequencies constant.
2. A bi-level optimization model is proposed to solve the problem of the macroscopic location of bus stops. The upper level of the model minimizes a cost function made up of user costs and operator costs and a lower level containing a modal split assignment model taking into account the influence of private traffic and congestion on public transport vehicles.
3. Optimal distances between stops obtained for each demand and congestion level indicate that very low demands produce wider spaces and make approximately 1200÷1600 metres (low bus stop densities). However, as demand increases, accessibility to public transport service should be increased and then spacing between bus stops drops to 360 metres. The distribution of bus stops is also homogenous in these high demand cases.
4. Sensitivity analysis of the relationship between different variables of the problem shows the relationship between public transport journeys and social cost, especially at high demands where any gain in passengers causing a reduction in congestion would be beneficial for the network as a whole.
5. Furthermore, at low demands, as operating costs increase, user costs decrease. However, as demand, and therefore congestion, increase, this relationship becomes inverted until it reaches a direct relationship between both costs.

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