CONTEXT SENSITIVE SOLUTIONS USING INTERVAL ANALYSIS

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Abstract. The term 'Context Sensitive Solutions' was adopted by the Federal Highway Administration in 1997 and is a synonym of flexibility that allows the designer to balance the safety, mobility and preservation of environmental resources. When the optimal use of design criteria produces an unacceptable solution, the correct application of design value outside the current guidelines with a particular attention to safety and legal risk is needed. In this case, a feasible alternative meets the purpose of design and is considered constructible and maintainable within social, economic and environmental constraints. Unlike in many other countries, road standards in Italy do not permit to work out a simple solution to the problem of flexibility. Specifically, one of the most debated subjects, concerning the items of the Italian Ministerial Decree (MD) 5/11/2001 confer with a designer the possibility of deviating from prescriptive obligations on condition to suggest some appropriate safety analysis. However, it does not indicate any methodology to objectively quantify removal effect from the reference values established by the rule. At this purpose, the paper suggests an analytical instrument for controlling design values outside the current guidelines applying a methodology based on interval analysis, a technique generally used for managing uncertain variables. The procedure applied to designing a planimetric curve has identified the most significant variables and produced some range in which they may be retained acceptable, though outside the limits of the geometric standard.

Keywords: flexibility, interval analysis, design standards, road design, safety.

1. Introduction

Generally, design concepts and values found in road standards in Italy and other countries are based on recognized practice and research (Brauers *et al.* 2008). The acceptable design values of any geometrical feature are established to assure, to the best knowledge possible, that the feature itself will not increase the risk of a crash and will contribute to make better traffic operations, capacity, constructability, maintenance, etc.

If an acceptable solution can be reached only with reference to design value marginally outside normal design criteria, it would be important that designers and a transportation agency had a right instrument to measure where, to what extent and under what conditions, eventually, to accept the proposed exception (Townsend *et al.* 2005; Crossett, Oldham 2005).

For this reason, documents contained into exception demand should include crash analysis, benefit cost analysis and rationale for deviation from the guidelines.

The Italian Ministerial Decree (2001) introduced for the first time in Italy a normative reference to geometric road design the structure of which receives some guidelines based on the AASHTO Green Book (A Policy on Geometric Design… 2004), British (Design Manual for Roads… 2002), Swiss and German standards (Lamm *et al.* 1999), especially as regards some consistency concepts of alignment with the management of geometric elements, the characterization of speed and the introduction of designing a speed diagram.

As regards the purely designing aspect, the application of standard principles implies some more recently signaled difficulties (Bosurgi *et al.* 2005, 2007). The results that emerged from the study have highlighted that some prescriptions, in particular for some road categories, are too much restrictive.

We refer to the methods for clothoid designing and, particularly to the observance of the minimum development of the residual arc that could be brought from a formal point of view to unjustified designing solutions incompatible with territorial pre-existences and costs.

Some road standards such as the Italian one do not help designers in managing the risk of accepting a design solution outside the typical ranges (Lambert, Turley 2005; Sander *et al.* 2006).

Therefore, a road designer has to find further and reliable information about other variables useful for assessing risk, like operating speeds, site crash history, roadside conditions, available pavement friction, etc. (Design Exception… 2003; Performance Measures… 2004; Flexibility in Highway Design 2004; Milton, Martin 2005; Paslawski 2008).

Concerning the above introduced information, the author proposes an analytical procedure for controlling the variables involved in road standards when one or more of these exceed the imposed limits. This methodology will be based on interval analysis and applied to designing a planimetric curve to highlight real advantages over traditional procedures.

2. Methodology

Designing horizontal curves is generally common among road standards established in different countries. The features of a circular bend include radius R, super elevation e, design speed V_d and side friction factor f (A Policy on Geometric Design… 2004). These models assume the vehicle system operates as a point mass with the vehicle centred in the lane and operating into the curve at a constant speed equal to the designed speed. This pattern would avoid the loss of control due to skidding that would occur if side friction demand exceeded pavement friction provided by the tire-pavement interface. The design value of f includes a substantial margin of safety against the loss of control due to skidding under the most available dry pavement conditions.

However, the design of a transition curve complicates the procedure considerably compared to the case of a simple circular arc.

2.1. Brief Notes on Italian Standards

The previous studies on a local rural road carried out by the author (Bosurgi *et al.* 2005, 2007) allowed reaching some results and are the base for the methodologies proposed in the present study. Particularly, relations between the value of circumference arc R and deflection angle α between two straights have been analyzed when the parameter A of the clothoid varies.

In order to operate the observance of standards, even the following value A^* has been evaluated, so that the development of the residual circumference allowed at least a distance of 2.5 seconds.

Therefore, if A_{st} is indicated as the highest value among the lowest ones required by standard criteria, the last value of A will have to respect inequality $A_{st} \leq A \leq A^\star$.

Obviously, in case of small angles of deflection, it is necessary to use very large radius R, otherwise, the development of too long clothoid branches would not guarantee the other verifications of parameter A. In fact, if results $A > A^*$, the clothoid would have a length that does not respect the lowest value of residual arc development ($t_{cir} > 2.5$ s).

A further condition of the new rule is about the calculation of parameter A, with the so called dynamic criterion.

In fact, the use of the highest design speed value is imposed to be deduced from the proper diagram, generally higher respect for V_d which characterizes the route on the circumference arc. In the numerical application, for the sake of simplicity, the maximum speed on the clothoid will be posed equal to V_{dcir} +10 km/h.

Condition $A = R$ is purely theoretic, because there is no possibility of using the excessive lengths of the clothoid that would be incompatible with the maintenance of an arc with circumference long enough to assure a run of 2.5 s at least.

In order to have the utmost observance of criteria for standards, a solution is obtained with very large radius and consequently, with very large V_d, unless the deflection angle between straight stretches is modified.

Different examined applications have indicated that the admissible zone, including value R reduces considerably at a decrease in the deflection angle between straight stretches.

The values lower than 40^c determine a moderate admissible zone that makes difficult the utmost observance of criteria for standards.

2.2. Short Notes on Interval Analysis

Interval Analysis (IA) was introduced at the beginning of the 20th century (Hayes 2003).

The first famous publication was work by Young (1931); still, this methodology had a strong pulse only twenty years later in work by Dwyer (1951), Warmus (1956) and Sunaga (1958, 2009). However, Moore (1967) developed more than all theoretical aspects deepening his studies on computer industry for more than forty years.

In IA, uncertain variables are characterized only by knowledge of the extremes of their field of existence. Also, the result of numerical calculations, therefore, will produce a range. (Dennis *et al.* 1998; Miao *et al.* 2009).

The paper deals with intervals having the following definitions:

$$x = [\inf(x), \sup(x)] = \{x \mid \inf(x) \leq x \leq \sup(x), \\ \inf(x), \sup(x), x \in \Re\}, \tag{1}$$

where: $\inf(x)$ denotes a lower limit to x; $\sup(x)$ denotes an upper limit to x.

Certainly, the uncertainty of the variable may be indicated by its lower and upper bounds of range or by means of the midpoint and its radius:

$$\mathrm{rad}(x) = w(x)/2; \tag{2}$$

$$\mathrm{mid}(x) = (\sup(x) + \inf(x))/2. \tag{3}$$

Other key features of these concern the concept of independence and extremes. Caused by independence, numerical values vary independently between intervals; calculations carried out at the ends of input variables, instead, lead to an output with the largest possible range.

Let $a = \left[\underline{a}, \overline{a}\right]$, $b = \left[\underline{b}, \overline{b}\right]$ be real compact intervals and ○ represents any operation as addition, subtraction, multiplication and division.

For example, for intervals *a* and *b*, operations can be defined by:

$$\left[a\right] \circ \left[b\right] = \left\{a \circ b \mid a \in \left[a\right], b \in \left[b\right]\right\},\tag{4}$$

where: $0 \notin [b]$ in case of division is assumed.

It is easy to prove that the set $I(\Re)$ of real compact intervals is closed with respect to these operations. What is even more important is the fact that $\left[a\right] \circ \left[b\right]$ can be represented by using only the bounds of $[a]$ and $[b]$. The following rules hold:

$$\left[a\right] + \left[b\right] = \left[\underline{a} + \underline{b}, \overline{a} + \overline{b}\right];\tag{5}$$

$$\left[a\right] - \left[b\right] = \left[\underline{a} - \overline{b}, \overline{a} - \underline{b}\right];\tag{6}$$

$$\left[a\right] \times \left[b\right] = \left[\min\left\{\underline{a}\underline{b}, \underline{a}\overline{b}, \overline{a}\underline{b}, \overline{a}\overline{b}\right\},\right.$$
$$\left.\max\left\{\underline{a}\underline{b}, \underline{a}\overline{b}, \overline{a}\underline{b}, \overline{a}\overline{b}\right\}\right]\tag{7}$$

if it is defined:

$$\frac{1}{\left[b\right]} = \left\{\frac{1}{b} \bigg| b \in \left[b\right]\right\} if \ 0 \notin \left[b\right],\tag{8}$$

then:

$$\left[a\right] \div \left[b\right] = \left[a\right] \times \frac{1}{\left[b\right]}.\tag{9}$$

Interval-valued functions follow from the interval arithmetic of two types (Neumaier 2001): interval extensions and united extensions (or true solution sets).

Interval extensions are functions where interval arithmetic is applied to calculate results.

United extensions are more computationally intensive and involve calculating fixed-point results with all possible combinations of variable interval endpoints. The disadvantage of interval extensions is that they can over expand the true solution sets of a function. This quality of interval extensions is unfortunate since both types of extensions guarantee the containment of all possible numerical results of the function giving inputs. Also, both extensions satisfy a property called inclusion monotonicity (given inputs, extension generates the widest possible bounds) which is similar to the extreme principle of interval arithmetic.

One of the strengths of IA is its ability to evaluate the whole range of values in one calculation that would take an infinite number of fixed-point calculations to produce (Carrizosa *et al.* 2004; Moerbeek *et al.* 2004; Qiu 2005; Mitrea, Tucker 2007).

This method provides easy deterministic implementation of the multiple state of design and produces the ranges of values for evaluation (Alefeld, Mayer 2000; Hargreaves 2002).

2.3. Application

The proposed procedure has been applied in the study on a transition curve. The aim of the plan is to evaluate the most critical parameters for fulfilling requirements determined by road standards in the most convenient way.

This methodology would respond to a precise request for FHWA:

- What is the degree to which a guideline is reduced?
- Will the exception affect other guidelines?
- Are there any additional features mitigating deviation introduced?

The variables treated with interval analysis are superelevation (*e*), deflection angle (α) and time for circular curve distance (t_{cir}).

Superelevation (*e*) characterizes the slope of a transversal section and for a local rural road and radii between 45 m and 437 m is always equal to 0.07. The deflection angle among the sides of two straight stretches directly influences the length of residual circumference. To assure respect for Italian Road Standards, a great value of α is needed, which is often inconsistent with the morphology of a territory. Time for distance represents time spent by the driver on the residual circular curve travelling at design speed V_d. Italian standards have established minimum time (2.5 seconds) and this prescription imposes great values of choosing deflection angles and radii *R*.

Although it is possible to choose other variables, however, it is not convenient to use design speed V_d. As for design speed exception, AASHTO (A Policy on Geometric Design… 2004) recommends that designers should not propose alternative design speed, because this variable is important for all features on the road. It will potentially result in unnecessary reduction in all speed-related design criteria rather than in only one or two features that led to the need for the exception.

In particular, we assigned some deviations (rad) from the nominal (mid) values of superelevation (*e*), deflection angle (α) and time employed to cover input variables of residual circumference (t_{cir}).

The nominal value coincides with the value assumed by the parameter of limit check that is a scenario where the values assigned to certain input variables allow no flexibility in the management of output values (Table 1).

Table 1. Limit check with Italian road standard

Trial 1	V_d = 70 km/h	
INPUT		
	mid	rad
q [%/100]	0.07	0.0000
α [cent]	47.0000	0.0000
t_{cir} [sec]	2.50	0.0000
OUTPUT		
	mid	rad
R [m]	178.00	0.00
A^* [m]	121.00	0.00
A_{st} [m]	121.00	0.00
Mar [m]	0.2829	0.00
SV_{cir} [m]	49.00	0.00

For example, the results achieved for the composition of a planimetric transition curve with the following input data give a rise to a single solution:

- design speed (V_d): 70 km/h;
- circumference radius (R): 178 m;
- the angle of deviation between sides of polygon (α): 47.0000 c;
- superelevation (e): 0.07 %/100;
- minimum time employed to cover residual circumference (t_{cir}): 2.50 seconds;
- design speed difference between circumference and clothoid start point (ΔV_d): 10 km/h.

Parameter A derived from regulatory limits (A_{st}) is 121 m, i.e. the designer might assume values equal to or greater than 121 m but lower than the radius of 178 m. Nevertheless, the values higher than 121 m result in choosing a clothoid of such a length that it does not allow a residual circumference coverage time of at least 2.5 seconds to be maintained as recommended in the norms. The designer is thus obliged to select value $A = 121$ m if s/he wishes to respect regulations.

The numerical formulations performed by assigning input variables in terms of the interval were considered to:

- rationalize the choice of acceptable ranges for output variables exceeding regulatory limits when these are deemed to be possible, for example following safety analyses;
- identify input parameters that affect output variables under examination the extent to which these may diverge from their nominal values.

The procedure will be applied to designing the planimetric curve in which the solution resulting from the imposition of the standard cannot be applied because of the presence of an obstacle (e.g. a building). As a result, we will study various provisions for alignment, slightly changing the angle of deviation between the two axes of the polygonal from 0 up to 6^c. This change will have an impact on dependent variables exceeding the limits of the standard. The most interesting aspect of the procedure is to obtain a solution to the designer's problem working on certain variables considered less critical than the others. That statement could derive, for example, from accident analysis (Kapskij, Samoilovich 2009).

3. Results

In order to test the procedure, we performed some numerical simulations, the results of which are summarized in the graphs and table below. In particular, the functions of A^* and A_{st} versus a variation on angle α have been evaluated to identify areas eligible for choosing parameter A. In Figs 1÷3, the cases with $R= 76$ m, 118 m and 178 m are reported.

The results obtained applying interval analysis regard the following variables (Table 2):

- R: circular curve radius;
- A^*: the clothoid parameter beyond which there is a possibility of having the minimum length of the arc is prescribed by the standard;
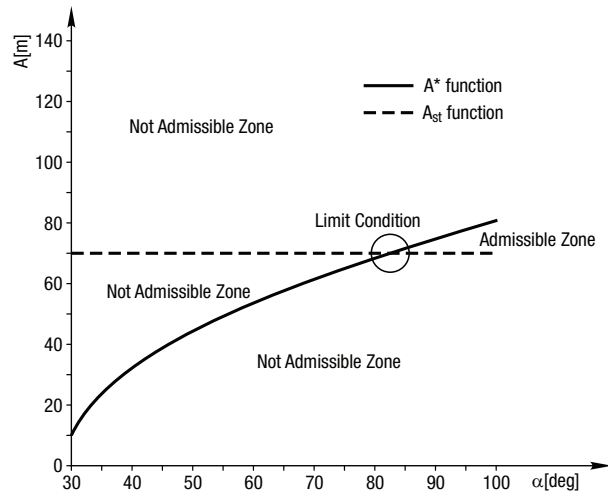

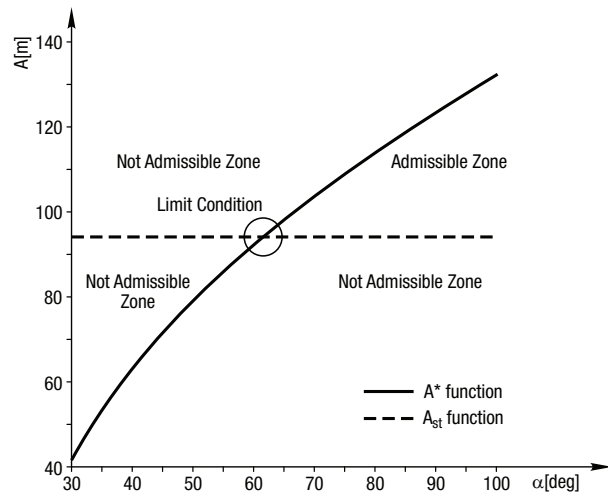Fig. 1. The function of A^* and A_{st} versus a variation on angle α when $R= 76$ m

Fig. 2. The function of A^* and A_{st} versus a variation on angle α when $R= 118$ m
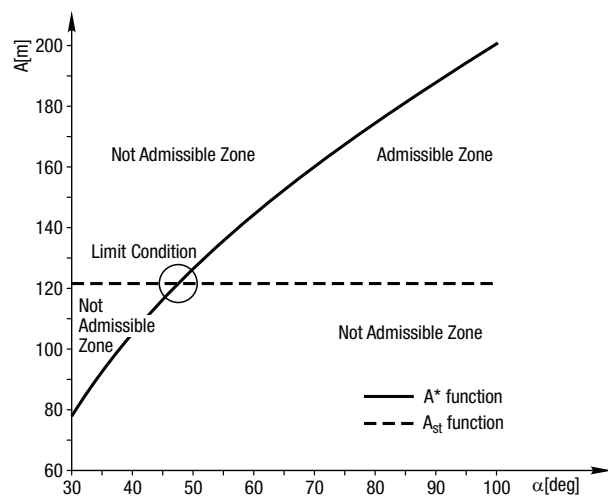

Fig. 3. The function of A^* and A_{st} versus a variation on angle α when $R = 178$ m

- A_{st}: the clothoid parameter derived from regulatory criteria;
- Mar: $A^* - A_{st}$ is the margin providing a possibility of selecting parameter A and completely satisfying regulations;
- SV_{cir}: residual length of the arc.

In practical design, if there is need for reducing deviation angle α between two sides of the polygonal road up to the value of 6^c (e.g. caused by a territorial constraint), the application of interval analysis in four different trials and setting variable α (in the form of <mid; rad>) amounted to <47; 0>, <47; 2>, <47; 4>, <47; 6> produced the results displayed in Tables 2 and 3.

In this simulation, variables (e) and (t_{cir}) have <rad> = 0.

The results produced in Tables 2 and 3 indicate that:
- there are no repercussions on R and A_{st};
- $A^* =$ <121; 12.3586> (Trial 4) means that in the worst combination possible $A^* = 121–12 = 109$ m. In this case, it is impossible to achieve $t_{cir} > 2.5$ seconds;

Table 2. Numerical processing based on interval analysis when $V_d = 70$ km/h, trials 1 and 2, α is the only input variable when rad $\neq 0$

Trial 1	Midpoint	Radius	Trial 2	Midpoint	Radius
INPUT			INPUT		
e [%/100]	0.07	0.0000	e [%/100]	0.07	0.00
α [cent]	47.0000	0.0000	α [cent]	47.0000	2.0000
t_{cir} [sec]	2.50	0.00	t_{cir} [sec]	2,50	0,00
OUTPUT			OUTPUT		
R [m]	178.00	0.0001	R [m]	178.00	0.0001
A^* [m]	121.00	0.0001	A^* [m]	121.00	4.1005
A_{st} [m]	121.00	0.0001	A_{st} [m]	121.00	0.0001
Mar [m]	0.2829	0.0001	Mar [m]	0.2136	4.1005
SV_{cir} [m]	48.99	0.0001	SV_{cir} [m]	48.99	5.5894

Table 3. Numerical processing based on interval analysis when $V_d = 70$ km/h, trials 3 and 4, α is the only input variable when rad $\neq 0$

Trial 1	Midpoint	Radius	Trial 2	Midpoint	Radius
INPUT			INPUT		
e [%/100]	0.07	0.00	e [%/100]	0.07	0.00
α [cent]	47.0000	4.0000	α [cent]	47.0000	6.0000
t_{cir} [sec]	2.50	0.00	t_{cir} [sec]	2.50	0.00
OUTPUT			OUTPUT		
R [m]	178.00	0.0001	R [m]	178.00	0.0001
A^* [m]	121.00	8.2151	A^* [m]	121.00	12.3586
A_{st} [m]	121.00	0.0001	A_{st} [m]	121.00	0.0001
Mar [m]	0.0045	8.2151	Mar [m]	–0.3482	12.3586
SV_{cir} [m]	48.99	11.1786	SV_{cir} [m]	48.99	16.7679

- consequently, $SV_{cir} =$ <49; 16> means that circumference development is 16 m lower than nominal measurement that guaranteed the coverage of the circumference arc in at least 2.5 seconds.

In practical terms, therefore, this departure really relates to reducing the residual length of the arc, i.e. $A = 121$ m must be used with the arc length of $SV_{cir} = 49–16 = 33$ m.

The proposed methodology, according to the established exception, permits to derive the range of accepting its dependent variables with single calculation.

Numerical results are summarized in Fig. 4 where the abscissa is a deviation from the nominal value of the input parameter (deflection angle α) and the ordinate represents a deviation from A^* and SV_{cir}.

However, to limit this methodology, such straightforward cases do not allow any appreciable advantages over traditional calculation methods. The greatest profit of this procedure is its ability to rapidly distribute the effects of an exception over more than one input variable.

Two further numerical simulations illustrate the above presented information. In the first case (Fig. 5), a fixed deviation from the superelevation rate (e) of 0.01 was introduced for every trial in conjunction with a deviation of variable (α) between 0^c and 6^c. As expected, the resulting values showed even higher deviation than nominal values when a single input variable was changed which suggests that:
- by establishing an admissible deviation from output variables, this exception can be distributed over one or more input variables simultaneously if necessary;
- if more than one input variable is involved, deviation will necessarily be lower than in case it was distributed over a single variable and its extent could be accurately calculated by the procedure.

The last simulation (Fig. 6) involves deviation from three input parameters such as deflection angle (α) (varying from 0^c to 6^c), superelevation (e) (kept at a constant 0.01) and the residual length of the arc (1 second).
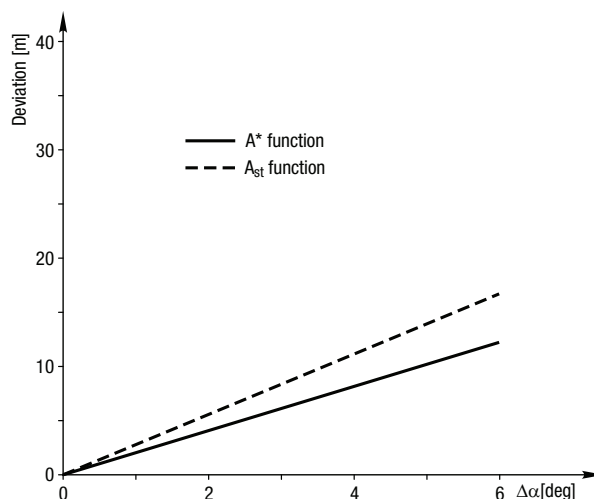

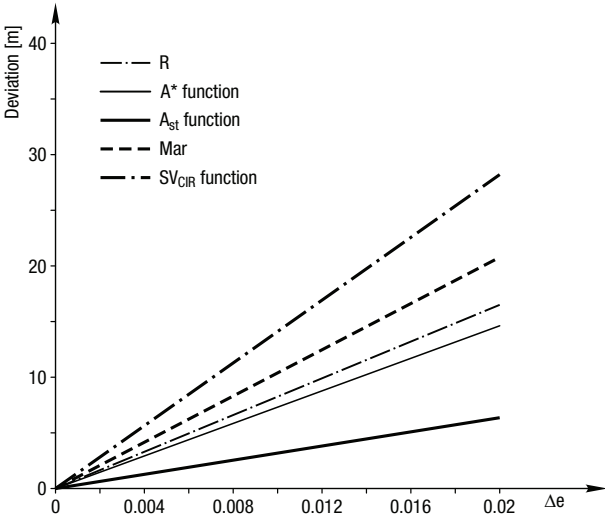**Fig. 4.** Relationship between $\Delta\alpha$, A^* and SV_{cir}

Fig. 5. Relationship between $\Delta\alpha$ and Δe with R, A^*, A_{st}, *Mar* and SV_{cir}, $\Delta e = 0.01$ was imposed
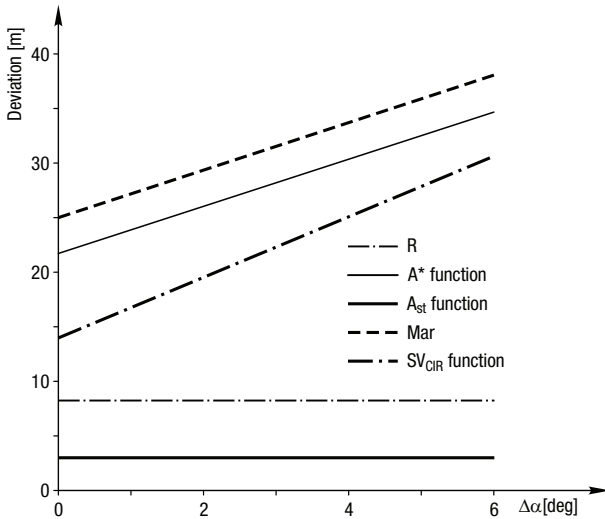
Fig. 6. Relationship between $\Delta\alpha$, Δe and Δt_{cir} with R, A^*, A_{st}, *Mar* and SV_{cir}, as input variables, $\Delta e = 0.01$ and $\Delta t_{cir} = 1$ was imposed

The graph illustrates the consistency of a number of output variables such as (R) and (A_{st}) and their dependence on other factors displaying higher values than those observed in the previous simulations.

This methodology permits the production of a synthesis table (Table 4) in which, once the maximum exception for the variable (SV_{cir}) has been assigned, it is possible to establish an acceptable range of dependent variables and, therefore imagine a range of scenarios presenting acceptable solutions.

To illustrate the procedure, for example, the choice of input and output parameters was introduced. Other variables and quite specific deviation values could be used if justified by suitable safety analyses.

Table 4. An example of the exception of different alternatives

	Exception of SV_{cir}=15 m		
	α [cent]	e [%/100]	t_{cir} [sec]
Alternative 1	0.0	0.011	0.0
Alternative 2	0.2	0.010	1.0
Alternative 3	0.4	0.010	0.0
Alternative 4	5.2	0.000	0.0

4. Discussion

This study was undertaken in response to the difficulties developers sometimes encounter when strict adherence to regulatory norms makes it impossible to find solutions that are respectful of the territorial context involved.

Any application for waiving norms addressed to the authorities responsible for granting designing permission should always include thorough preliminary accident analysis in order to distinguish variables that will allow no flexibility from those the ranges of which could safely be slightly wider than regulations prescribe.

To this end, a methodology based on interval analysis able to satisfy a number of requirements has been proposed and thus perform the following functions:

- derive, in a few simple analytic steps, new ranges of dependent variables from those identified by means of accident analysis;
- quantify how much waiving will be required;
- check which variables are affected and to what extent;
- facilitate the quantification of further risk before and after analyses and introduces mitigating elements.

When designing roads, there are some situations that the engineer can solve in different ways considering great flexibility, i.e. numerous solutions. On the other hand, boundary conditions are so restrictive that involve incompatibility with the standard. For example, Fig. 1 represents the case of a transition curve when $R = 76$ m and the solution fully respectful of the standard exists only with great deviation angles between the two sides of the polygonal (low admissible zone). If radius increases, the task of the designer is easier, as shown in Figs 2 and 3. However, it is not always easy if important spatial constraints exist.

Therefore, there is need to 'force' the limits imposed by the rule, provided however, by evaluating effects on dependent variables from the parameter you want to change. In this case, applying the interval analysis technique to (e), (α) and (t_{cir}) was an interesting point. In particular, the analyst may be interested in managing variation in one of these variables (for example, recognized critical analysis of an accident) and, conversely, exacerbate the range of the others two.

As combinations can be numerous, the results presented in this article have been limited due to reasons of synthesis (Figs 4÷6).

These considerations suggest that, with difficulty, rules indicate precise intervals to be given to variables, as this will depend greatly on the environmental context in which they are applied.

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