INVESTIGATION OF THE CONSTRUCTION OF PROBABILISTIC LOW CYCLE FATIGUE DESIGN CURVES AT STRAIN CYCLING

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Abstract. Probabilistic methods are based on the use of statistical data about mechanical characteristics and lifetime at cyclic loading of a material. In this investigation statistical low cycle fatigue tests were carried out and mechanical characteristics of three materials (steel of grades 15X2MFA and 45 and aluminium alloy D16T1), representing all possible variations of cyclic properties (fatigue, quasi – static and intermediate mode of fracture) were determined experimentally. Based on the obtained data the analysis of design curves of equal fracture probability for steels 15X2MFA and 45 and aluminium alloy D16T1 at strain low cycle loading was performed and the design curves were compared with the experimental ones.

Keywords: low cycle fatigue structural materials, strain cycling.

1. Introduction

Aiming to achieve the best performance (capacity, productivity, speed, etc.) at minimal metal consumption a lot of equipment conforming to high requirements of safety and reliability operates under the conditions of high stresses, resulting in some structural elements elastic – plastic static or cyclic deformations.

Under such conditions the problems of the development of reliable and efficient structures rise in importance. The probabilistic methods of the strength and durability assessment of various structures, aiming to improve their reliability and safety, are continuosly increasing application. The methods first of all are based on the use of statistical data of mechanical characteristics at cyclic loading of a material.

The probabilistic evaluation of resistance to low cycle fatigue loading and fracture of the structural elements subjected to high cyclic operational load arising due to changes of the operational pressure and temperatures is paramount for the reliable use of transport (automotive, railway, sea) and energy (e.g. nuclear reactors) equipment which total specified life is 10^3-10^4 loading cycles.

The history of the probabilistic approach to the structure design and the analysis of estimated materials performance reckon more than 50 years. Great contribution to the development of probabilistic computation methods, probabilistic substantiation of allowable stresses and strength safety margins in the static and cyclic strength calculations is made by the outstanding investigators such as W. A. Weibull, A. M. Freudenthal, E. J. Gumbel, K. Iida, H. Inoue, S. V. Serensen, M. N. Stepnov, J. V. Giacintov, V. P. Kogaev and others [1–7].

In general up to recent time systematic probabilistic evaluation was used in the assessment of statistical and fatigue properties investigation data. Such assessments for low cycle fatigue data are not numerous.

Aiming to develop the base for low cycle fatigue fracture evaluation the method of experimental investigation and construction of probabilistic low cycle fatigue estimation curves is proposed and the comparison of these curves with the experimental data is performed.

2. Substantiation of materials selection

The analysis of the literature sources shows that due to statistical nature of a material itself and its fracture process the scatter of material properties takes place. The scatter, probably, depends on the cyclic properties of the material and on the fracture character at low cycle loading. This fracture can be defined as fatigue fracture, quasi - static and intermediate

between fatigue and quasi - static fractures.

Aiming to analyze statistical distribution of mechanical characteristics and low cycle loading parameters, three materials having contrasting properties at cyclic loading: softening steel 15X2MFA, weakly softening (practically stable) steel grade 45, and strongly hardening aluminium alloy D16T1, were used. Tests were carried out at tension compression strain cycling at constant strain ratio – 1.

Material of the same welting was used for the specimens. Steel 15X2MFA specimens were cut from a rolled 120 mm diameter bar in such a way that rolling direction coincided with the specimen axis. Specimens of steel grade 45 and aluminium alloy D16T1 were cut from 50 mm diameter bars. Rollings of 15X2MFA steel and bars of aluminium alloy D16T1 were subjected to heat treatment. Steel 15X2MFA was heated to 1000 ^{0}C, quenched in oil and twice tempered at 700 ^{0}C during 70 hours. The standard hardening and tempering procedure was used for aluminium alloy D16T1.

Chemical composition and mechanical characteristics of the tested materials are presented in Tables 1 and 2.

The chosen material represents the main varieties of cyclic properties: cyclic hardening, cyclic stabilization and cyclic softening (Fig 1). That is why the conclusions based on the obtained experimental data and theoretical analysis can be used for the evaluation of various structural materials, operating under low cycle loading conditions.

3. Methodology and program of monotonic and low cycle loading tests

Standard 10 mm diameter and 23 mm length of deformable part specimens were used for statistical low cycle fatigue tests. Fractured during cyclic loading specimens were used as work pieces for machining of monotonic tension specimens. These specimens have $d = 5$ mm diameter working part which length is $5d$. This part of the specimens is made from fractured during cyclic loading specimen part diameter
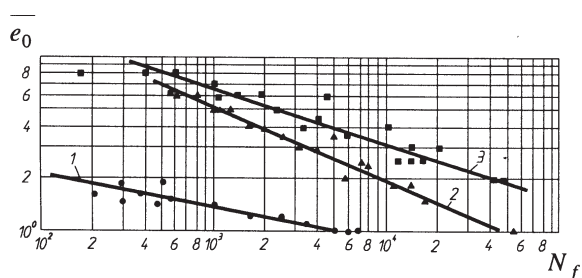

Fig 1. Low cycle fatigue curves at strain cycling: 1 – D16T1; 2 – 15X2MFA; 3 – 45

of 17,4 mm which during cycling was subjected to elastic loading only. Aiming to prevent elastic – plastic bending of the specimen during machining due to the action of cutting forces, the cutting depth was decreased with the decrease of the specimen diameter. For the same purpose machining was performed with high sped steel cutting tools which geometry was chosen aiming to minimize cutting force. Finishing of the specimens was performed with five abrasive papers on a lathe.

Loads applied and numbers of specimens tested at every of these load values are presented in Table 3. The table shows that statistical experiment was carried out at three load levels. It is recommended [8–10] building a low cycle fatigue curve to test at least 15 specimens at various evenly distributed over the range loads. This recommendation is based on the researches [3, 10], which advise to use at least 20–25 specimens at one load level for statistical investigation. Such amount of specimens enables to obtain stable enough distribution characteristics which change during the repeated tests is negligible. In our investigation 20 specimens were tested at each load level. In some sources [10, 11] it is shown that in the intermediate zone the increased scatter in comparison with quasi - static and fatigue fracture zones was observed. That is why in this zone the number of specimens was increased. Three strain levels at strain cycling are evenly distributed over the range (Table 3). At strain cycling on all strain range levels fatigue fracture was obtained. Despite of that, in the middle level the number of specimens was increased for all materials tested in order to clear up the effect of

Table 1. Chemical composition of the tested materials, %

Material	C	Si	Mn	Cr	Ni	Mo	V	S	P	Mg	Cu	Al
15X2MFA	0,18	0,27	0,43	2,7	0,17	0,67	0,3	0,019	0,013	-	-	-
45	0,46	0,28	0,63	0,18	0,22	-	-	0,038	0,035	-	-	-
D16T1	-	-	0,7	-	-	-	-	-	-	1,6	4,5	9,32

Table 2. Mechanical characteristics of the tested materials

Material	e_{pr}, %	σ_{pr}, MPa	σ_y, MPa	σ_u, MPa	S_k, MPa	Ψ, %
15X2MFA	0,2	280	400	580	1560	80
45	0,26	340	340	800	1150	39
D16T1	0,6	290	350	680	780	14

Table 3. Statistical low cycle fatigue tests program

Material	Loading level, $\overline{e_0}$	Number of specimens
15X2MFA	1,8	20
	3,0	40
	5,0	20
45	2,5	20
	4,0	100
	6,0	20
D16T1	1,0	20
	1,5	40
	2,0	20

specimen amount on statistical characteristics of both stress – strain diagrams and lifetime. The strain rate (4 mm/min and 20 mm/min) constant for all specimens was held.

4. Mathematical analysis of the construction of probabilistic low cycle fatigue design curves for structural materials at strain cycling

The purpose of strain cycling tests is to define material fracture characteristics at elastic – plastic cyclic loading. For the assessment of material strength the Coffin – Manson equation [11, 12] can be used

$$e_p N_f^m = C_v ,\qquad (1)$$

where e_p is plastic strain range of the cycle; N_f is the number of cycles to fracture; m and C_v are constants which according to L. Coffin data for great majority of materials equal to 0,5 and $\frac{1}{2}\ln\frac{100}{100-\psi}$ correspondingly.

Equation (1) reflects linear relationship between plastic strain and the number of cycles to fracture in the logarithmic coordinates $\lg e_p - \lg N$. Taking into account the experimental fact that plastic strain during strain cycling changes and only for cyclically stable materials it is constant, the authors recommend to use value C_ψ, that corresponds to 50 % of loading cycles to fracture, i.e. when the process of stabilization starts.

Based on Langer equation the strain amplitudes and relative stress at constant strain amplitude symmetrical cycling are expressed [13]:

$$e_a = \frac{1}{4e_{pr}}\frac{1}{N^m}\ln\frac{100}{100-\psi}+0,4\frac{\sigma_u}{Ee_{pr}},\qquad (2)$$

$$\sigma_a^* = \frac{1}{4}\frac{E}{N^m}\ln\frac{100}{100-\psi}+0,4\sigma_u ,\qquad (3)$$

where m is a constant equal to 0,5 $\sigma_u < 687$ MPa;

e_{pr} is the strain corresponding to proportionality limit stress; σ_a^* is relative elastic ($\sigma_a^* = e_0 e_{pr}E$).

The plastic strain is calculated according to the formula:

$$e_0 = \frac{0,5\ln\dfrac{100}{100-\psi}}{\left(4N^{0,5}\right)}+\frac{\sigma_u}{E(4N)^{0,05}} .\qquad (4)$$

According to the data presented in [14], for majority of metals equation (1) can be rewritten in the form:

$$\varepsilon N_f^{\alpha_{1p}} = C_{1p} .\qquad (5)$$

Differently than in Coffin equation here $\alpha_{1p} < m$ and $C_{1p} < C_\psi$. Constants α_{1p} and C_{1p} can be defined from material mechanical characteristics

$$\alpha_{1p} = 0,17+0,55\psi\frac{\sigma_y}{\sigma_u} ,\qquad (6)$$

$$C_{1p} = 0,75\alpha_{1p}\ln\frac{100}{100-\psi} .\qquad (7)$$

Up to recent time strain cycling design curves were built according to (1)–(7) or analogous equations, that used mechanical characteristics corresponded to 50 % probability. Thus, it was assumed that design curves describing low cycle fatigue resistance corresponded to the same (50 %) fracture probability.

Probabilistic design curves of low cycle fatigue should be made using material mechanical characteristics determined by standard tests. Ultimate tensile strength and plasticity characteristics of a material are used in the calculation of equation. (5) coefficients α_{1p} and C_{1p}. It should be noticed that physical relationship between the characteristics of strength and plasticity is not unique, therefore, when ultimate tensile strength is determined with certain probability (e.g. 95 %) from the statistical data file, the relative reduction will be determined from the same data file with different probability. That is why in this investigation the assumption is made that there is equal probability of all characteristics determination from the same data file.

In the course of this investigation the probabilistic values of mechanical characteristics were determined. This made it possible to make low cycle fatigue design curves corresponding to various fracture probabilities. Probabilistic values of mechanical characteristics of softening during cyclic loading steel 15X2MFA, stable steel grade 45 and hardening aluminium alloy D16T1 are presented in Table 4.

By means of Eqs. (2), (4) and (5) and using values of equal probability mechanical characteristics, the low cycle fatigue at strain cycling curves corresponding 1, 10, 30, 50, 70, 90 and 99 % fracture probability in the coordinates $\lg e_0 - \lg N_c$ for steel 15X2MFA were constructed. Calculation according to Eqs. (2) and (4) gives slightly different in respect to both the slope and the scatter band probabilistic curves of low cycle fatigue. Numerical relation between the lives calculated from 1 and 99 % probability curves slightly depends on strain range and for the curves calculated according to relationship (2) average is about 3,2, and for the curves (4) – about 3,3.

The construction of the probabilistic low cycle fatigue curves by means of Eq. (5) for steel 15X2MFA gives a satisfactory result because these curves at lives about 200–400 cycles intersect and at lives more than 400 cycles these curves are arranged in the reverse order, i.e. the minimum lifetime is obtained at 99 % of fracture probability and the maximum – at 1 % frac-

ture probability. It is because coefficients α_{1p} and C_{1p} depend on mechanical characteristics of a material. As it follows from Eqs. (6) and (7), probabilistic value of the coefficient α_{1p} mainly depends on relative reduction of cross-section area because probabilistic ratios σ_{pr}/σ_u differ negligibly and vary from 0,41 (curve of 1 % probability) to 0,56 (curve of 99 % probability). Constant C_{1p}, which depends on α_{1p} and on the reduction of cross-section area varies between the limits 0,42 (1 % probability curve) and 0,96 (99 % probability curve). Due to these variation of coefficients α_{1p} and C_{1p} the probabilistic curves are arranged in the way shown in Fig 2.

Low cycle strength of conforming to high requirements of safety and reliability parts and structural elements of transport and power equipment is often calculated according to the relationships constructed in relative coordinates. Low cycle fatigue probabilistic curves for steel 15X2MFA, therefore, were constructed in relative coordinates using Eqs. (2), (4) and (5).
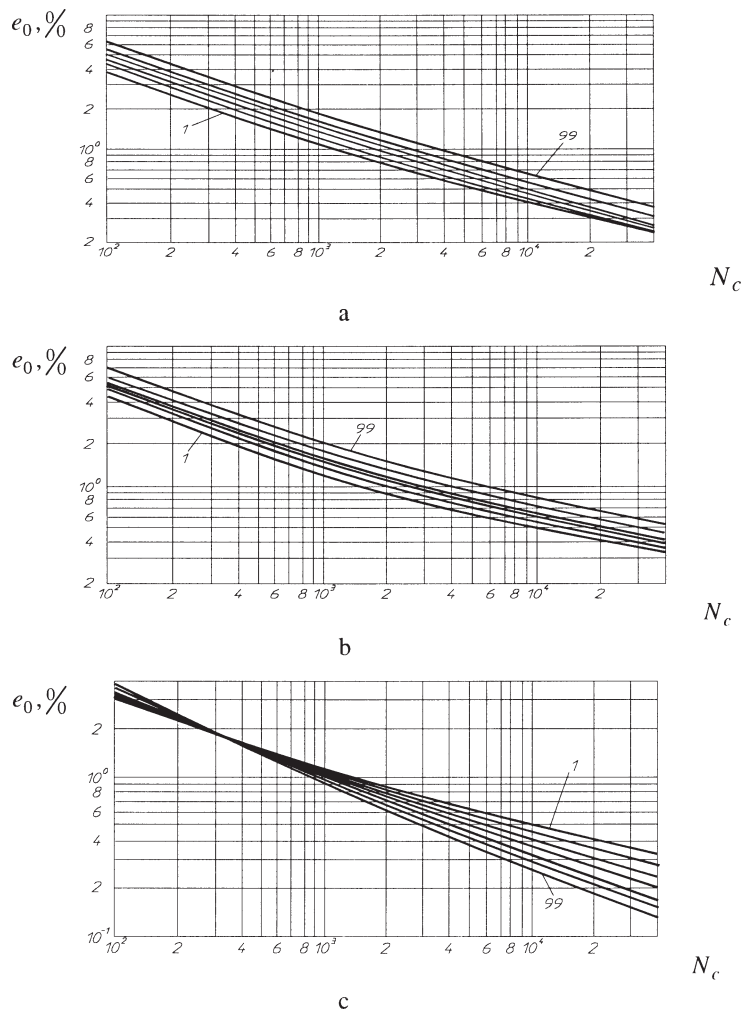
Fig 2. Low cycle fatigue curves at strain controlled loading of steel 15X2MFA according to parameter of fracture probability in absolute coordinates: a, b, c – curves calculated from Eqs. (2), (4) and (5) correspondingly. Numbers by the curves mean 1 and 99 % probability

For this purpose strain amplitudes of 1 % probability curve were divided to strains e_{pr}, corresponding to 1 % probability. Accordingly, the strains of 10 % probability curve were divided to e_{pr}, corresponding to 10 % probability, etc., etc. In such a way the constructed low cycle fatigue curves in coordinates $\lg e_0 - \lg N_c$ are shown in Fig 3 (curves a, b, c).

Probabilistic low cycle fatigue curves in the relative coordinates give a distorted view of the relative arrangement of the probabilistic design curves for steel 15X2MFA. Such a reverse order of low cycle fatigue probabilistic curves arrangement is related to very big scatter of the strain e_{pr} values in comparison with other mechanical characteristics, e.g. the reduction of cross-section area which predetermines lifetime (Table 4).

Difference in the scatter of strains e_{pr} and other

material characteristics can be explained, probably, by strong dependence of the strain on heat treatment, prestrain in the course of machining, precision of the test and other factors. The effect of these factors on the other material characteristics is incomparably smaller. The analysis performed allows to conclude that for the construction of low cycle fatigue probabilistic curves in the relative coordinates probabilistic value of strain e_{pr} should not be used because the distortion of a real picture of the curves arrangement is possible (Fig 3, curves a, b, c).

In order to determine real arrangement of low cycle fatigue probabilistic design curves at strain controlled loading for steel 15X2MFA in relative coordinates the strains corresponding to certain probability curve were divided to the mean value of e_{pr}. In such case, as it follows from Fig 4 (curves d, e, f), the ar-
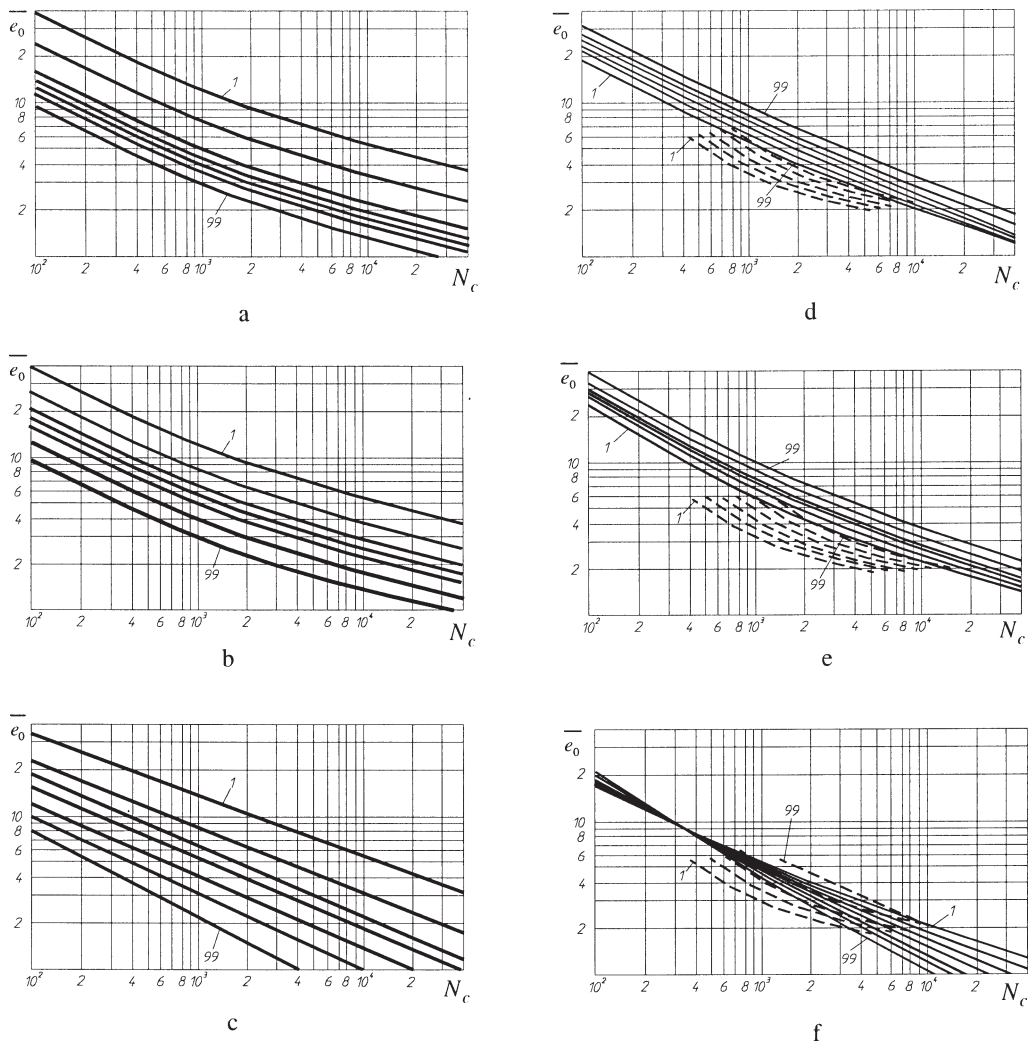
Fig 3. Low cycle fatigue design curves at strain controlled loading of steel 15X2MFA according to parameters of fracture probability in relative coordinates: a, b, c – curves calculated from Eqs. (2), (4) and (5) correspondingly; strains e_{pr} determined according to corresponding probability curve; d, e, f – curves calculated from Eqs. (2), (4) and (5) correspondingly; strains e_{pr} corresponding to 50 % probability of the mechanical characteristics of the material. Numbers by the curves mean 1 and 99 % probability. Experimental curves for corresponding probabilities are shown in dashed lines

Table 4. Probabilistic values of the mechanical characteristics of steel 15X2MFA and 45, and aluminium alloy D16T1

Material	Characteristic	Probability, %						
		1	10	30	50	70	90	99
15X2MFA	σ_{pr}, MPa	300	340	370	400	430	475	535
	σ_u, MPa	500	530	560	580	600	640	680
	Ψ, %	74	75	79	80	82	85	90
	e_{pr}, %	0,09	0,13	0,17	0,2	0,245	0,32	0,475
45	σ_{pr}, MPa	220	265	300	340	360	420	500
	σ_u, MPa	620	700	750	800	850	900	1020
	Ψ, %	28	32	37	39	42	47	54
	e_{pr}, %	0,14	0,18	0,225	0,26	0,3	0,36	048
D16T1	σ_{pr}, MPa	260	300	320	350	370	405	460
	σ_u, MPa	580	620	650	680	700	750	800
	Ψ, %	9,5	11,3	12,8	14	15,5	17,5	21
	e_{pr}, %	0,46	0,52	0,56	0,6	0,64	0,7	0,78

a

b
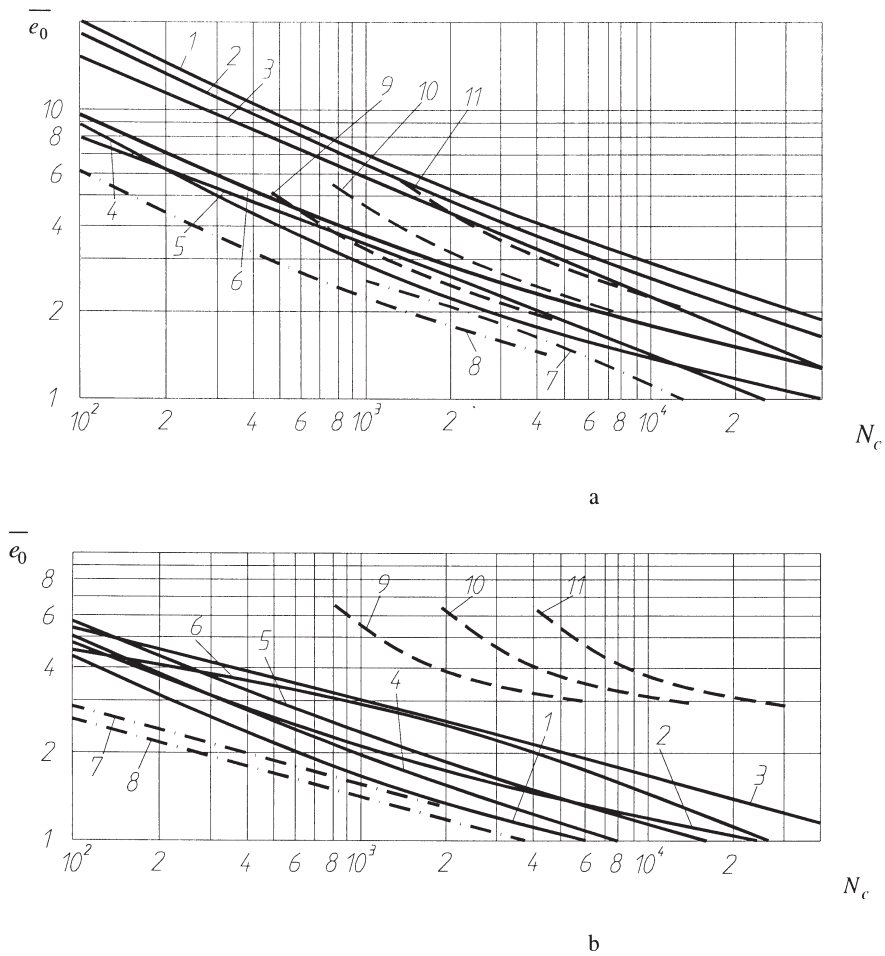
Fig 4. Comparison of low cycle fatigue design curves at strain cycling with experimental ones for steel 15X2MFA (a) and 45 (b): 1–3 – curves constructed according to Eqs. (2), (4) and (5) for guaranteed mechanical characteristics; 4–6 – the same for characteristics according [5]; 7, 8 – design curves with safety factors n_e =2 and n_N =10 correspondingly; 9–11 – experimental curves, corresponding to 1, 50 and 99 % fracture probability

rangement of the curves very slightly differs from that in absolute coordinates (Fig 2). The ratio of durabilities of 99 % and 1 % probability design curves constructed according to Eqs. (2) and (4), negligibly depends on the strain and in average for the curves constructed according to Eq. (2) is equal to 2,9 and for those constructed according to Eq. (4) – 3,2.

5. Verification of probabilistic low cycle fatigue design curves of structural materials at strain cycling

Verification of probabilistic design curves was performed by the comparison with experimental steel 15X2MFA curves of the same probability. The slope and scatter band size of the experimental curves approximately coincided with those of design curves constructed according to Eqs. (2) and (4). But the experimental curves are arranged below the design curves. For example, at short lifetime 99 % experimental curve coincides with 50 % design curve constructed according to Eq. (2), and 50 % experimental curve – with 1 % curve. A little worse correspondence is obtained in the region of long lifetime for the curves constructed according to Eq. (4). In this case 99 % experimental curve lies below 1% design curve in the region $N_c > 7 \cdot 10^3$ it coincides with 10–20 % design curve. Probabilistic design curves constructed according to Eq. (5), practically coincide with experimental ones. Analogous analysis was performed for steel 45 at strain controlled cycling. The equal probability curves were constructed according to Eqs. (2), (4) and (5) using probabilistic values of mechanical characteristics. The arrangement of probabilistic design curves constructed according to Eqs. (2) and (4) and plotted in absolute coordinates $\lg e_0 - \lg N$ was similar in respect to both the slope and scatter band to that of curves constructed according to Eq. (5). The increase of coefficient α_{1p} with the increase of fracture probability was observed. But in this case due to a larger range of coefficient α_{1p}, the probabilistic curves are arranged in the usual order, i.e. 1 % curve yields minimal and 99 % curve – maximal lifetime.

Lifetime ratio of 99 % and 1 % probabilistic curves for steel 45 depends on a strain level. The performed analysis shows that for the curves constructed according to Eqs. (2), (4) and (5) the lifetime ratio for 99 % and 1 % curves for the strain amplitude $e_0 = 0,9$ % makes correspondingly 7,3; 8,2; 10,6 and for $e_0 = 0,4$ % - 4; 6,7; 20,7.

When probabilistic design curves for steel 45 are constructed in relative coordinates $\lg e_0 - \lg N_c$, the use of probabilistic values distorts both the real picture of the curve arrangement and the lifetime scatter calculated according to the above mentioned equations. At the strain amplitude $\overline{e_0} = 4$ lifetimes ratio of

1 % and 99 % probabilistic curves constructed according to Eqs. (2), (4) and (5) makes correspondingly 2,8; 3,8; 8,3. At $\overline{e_0} = 2$ the corresponding values are 4; 6,7; 20,7.

In order to obtain actual arrangement of probabilistic design curves of steel 45 in relative coordinates strains e_0 were divided by average value of e_{pr}, just as it was done for steel 15X2MFA. Due to such interpretation of the data related to the arrangement and slope, the probabilistic design curves obtained are similar to the curves in absolute coordinates. Lifetime ratio of 99 % and 1 % curves similar to that of curves in absolute coordinates depends on a strain level: at $\overline{e_0} = 4$ lifetime ratio for curves (2), (4) and (5) makes 7,1; 7,6; 10,7 correspondingly, and at $\overline{e_0} = 2$ – correspondingly 8,4; 11,7; 5,3 (Fig 4, b). Probabilistic design curves for steel 45 are arranged below 1 % probabilistic experimental curve. The equation (5) yields the results nearest to the real situation in comparison with all other equation used for the construction of probabilistic low cycle fatigue design curves for steel 45. The construction of the probabilistic curves for steel 45 according to Eqs. (2) and (4) is possible, but in this case there is noticeable difference between calculated and experimental data.

Low cycle probabilistic design curves for aluminium alloy D16T1 at strain controlled loading were constructed according to Eq. (5). The procedure used was the same as that applied for steel i.e. the curves were plotted in absolute and relative coordinates using probabilistic mean value of e_{pr}. The analysis of the scatter band showed that lifetime ratio of 99 % and 1 % probabilistic curves depends on the strain level. The lifetime ratio at strain amplitude $e_0 = 0,3$ % is 37, and at $e_0 = 0,18$ % - 24. The increase of the slope of probabilistic design curves with the increase of fracture probability was observed. This feature similarly as for steels 15X2MFA and 45, is influenced by the scatter of relative reduction of cross-section area.

In calculation the use of the relative strain $\overline{e_0}$ determined according to probabilistic values of e_{pr} results in a more narrow lifetime scatter band. In this case lifetime ratio of 99 % and 1 % probabilistic curves at strain amplitude $\overline{e_0} = 4$ obtained 3,3, and at $\overline{e_0} = 3$ – 2,7. The slopes of the probabilistic design curves in relative coordinates are reduced in comparison with the curves in absolute coordinates. However, the slope of the curves increases with the increase of fracture probability.

It depends on the strain level. The lifetime ratio at strain amplitude $e_0 = 0,3$ % is 37, and at $e_0 = 0,18$ % - 24. The increase of the slope of probabilistic design curves with the increase of fracture probability was observed. This feature similarly as for steels 15X2MFA and 45, is influenced by the scatter

of relative reduction of cross-section area.

In the same way as for steel 15X2MFA and 45, the mean strain value e_{pr} was used for strain e_0 calculation. This resulted in different arrangement of the probabilistic design curves. In this case lifetime ratio of 99 % and 1 % probabilistic curves is close to hat of the experimental probabilistic curves constructed in absolute coordinates i.e. at the strain $\overline{e_0} = 4$ this ratio is 32, and at $\overline{e_0} = 3$ it is 25,5.

In Fig 4 the experimental low cycle fatigue at strain controlled cycling curves corresponding to 1, 50 and 99% fracture probability is shown . The curves are constructed according to Eqs. (2), (4) and (5) using mechanical characteristics presented in Table 4 and in source [15, 16]. Fig 4. also presents low cycle fatigue curves calculated applying safety factors used in power equipment $n_e = 2$ for strain amplitude and $n_N = 10$ for the number of cycles. The latter curves were constructed by means of low cycle fatigue curves calculated using guaranteed mechanical properties and relationships, most precisely describing experimental lifetime, i.e. Eq. (2) for steel 15X2MFA, and (5) for steel 45.

Low cycle fatigue curves for steel 15X2MFA (Fig 4, a) constructed according to Eqs. (2), (4) and (5) using guaranteed mechanical characteristics are located over the experimental 99 % fracture probability curve. The same curves constructed using mechanical characteristics according to [5], are located in the band between 1 % and 50 % probability curves. Such arrangement of the curves should be expected because the guaranteed mechanical properties of 15X2MFA steel are close to experimental values of these characteristics – and mechanical characteristics according by [15] have great scatter. Due to such high location of the low cycle fatigue design curves in relation to experimental ones, the curves constructed accounting safety factors $n_e = 2$ and $n_N = 10$ high enough are located. Low cycle fatigue curve constructed using factor $n_e = 2$ practically coincides with experimental 1 % fracture probability curve and the curve constructed for safety factor $n_N = 10$ is arranged below.

A different picture was observed for steel 45 (Fig 4, b). The design curves constructed using both guaranteed mechanical characteristics and those according to [5] are arranged below the experimental ones. But this feature is the result of poor agreement of the design curves determined from Eq. (5) with experimental ones.

The performed analysis of relative arrangement of the design and experimental probabilistic low cycle fatigue curves revealed that the construction of the curves according to general analytical relationships for certain materials can lead to essential errors. That is why it is advised during designing to have at least

the experimental 50 % fracture probability curve by means of which it is possible to access reliably strength and lifetime of the structure or its element under design.

Thus, it is advised to refuse from the construction of probabilistic curves in relative coordinates using probabilistic elastic limit strain in the case when the scatter of this characteristic essentially exceeds the scatter of other strength and plasticity characteristics. The relationships (2), (4) and (5) presented are not universal because for steel 15X2MFA better agreement of the calculation data with experimental is obtained using Eqs. (2) and (4), and for steel 45 – using Eq. (5).

6. Conclusions

1. The analysis of the calculated and experimental low cycle fatigue curves constructed according to the fracture probability parameter revealed that the use of analytical relationships for lifetime assessment of structures may lead to the error decreasing safety margin.

2. It is recommended to use the experimental 50 % fracture probabilistic curve for the reliable strength and lifetime assessment of power structures.

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